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Multiple discrete continuous choice modeling through additively and non-additively separable functional forms

To apply for a Master's degree in Environmental and Natural Resources Economics

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Abstract

This paper compares welfare measures for Multiple Discrete-Continuous (MDC) choice models with Additively Separable Utility (ASU) and Non-Additively Separable Utility (NASU) functions. The uncommonly used NASU approach offers flexibility for integrating complementarity and substitution patterns among alternatives. Nevertheless, welfare measures calculation is more complicated and it has rarely discussed in the literature. In contract, ASU models allow us to calculate welfare measure more easily. In this article, we use the generalized ASU and NASU functions and the minimization of the expenditure function to estimate welfare measures. Our empirical analysis uses data from the Canadian Nature Survey which includes information regarding the number of days dedicated to different recreational activities. Our results show statistically significant differences in welfare measures for changes in prices. Welfare measures from NASU function are lower than those from a ASU function.

Keywords: choice modeling, discrete/continuous, welfare measure

Contents

List of Tables

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Chapter 1

Introduction

1.1 Introduction

The study of consumer behavior generally requires the understanding of two types of decisions. On the one hand, individuals must decide which goods to acquire and which goods not to acquire (discrete decision), on the other hand, for which goods to acquire, individuals must decide how much of these will acquire (continuous decision). For example, an individual must decide whether to visit parks A and/or B, and then how many visits to park A and/or B. In principle, researchers are interested in explaining both decisions in an integrated utility model and not each as a mutually exclusive decision of the other.

Initially researchers have used simple discrete choice models to understand consumer choice preferences (discrete decision), these models assume that the individual chooses only one alternative within a set of available alternatives (for example multinomial logit, nested logit (McFadden, 1977) and mixed logit (Train, 1998; 2003)). However, individuals commonly face situations where they can choose more than one alternative within the set of alternatives. Additionally, the individual chooses a continuous amount of the alternatives he chooses (continuous decision), this has been commonly generalized as multiple discrete-continuous (MDC) choice. These kinds of decisions are commonly observed in the behavior of individuals, for example, in the environmental context, consumers should choose what recreational activities they prefer to develop (fishing, hiking, hunting, etc.) and how many times they prefer to develop them (twice fishing, never hiking, once hunting, etc.)

Commonly modeling MDC choices is underpinned by the microeconomic problem of maximizing consumer utility. Among the most used approaches is the multiple discrete-continuous extreme value model (MDCEV) of Bhat (2008), which is the generalization of the multinomial logit model (MNL), one of the main advantages of this formulation is the closed form of the expression of probability.

This chapter first presents a framework for understanding and modeling the MDC model. Then, the main guidelines of this investigation are presented (justification, objectives, purpose, and limitations).

1.1.1 MDC choice models

The first efforts to model the MDC choice are made through the use of simple choice models, the approaches used were: bundles, stitching, and stream. The first approach begins by identifying all sets of elementary alternatives and then, each set is used as a composite alternative in a single model of choice. The second approach begins with the union of simple discrete choice models, which model multiple discrete decisions through correlation methodologies between univariate utility maximization problems. Finally, the third approach models the multiple discrete choice decision as to the result of future consumption decisions. However, these three approaches are not based on a utility maximization model.

Early efforts to model the structure of MDC through an underpinning in preference theory were referred to as the first-order conditions (FOC) approach of Karush-Kuhn-Tucker (KKT) for restricted maximization of random utility, an approach attributed to Hanemann et al. (1978) and Wales and Woodland (1983). This approach begins with the formulation of a utility function whose arguments include the level of consumption of each product and also the quality and other attributes of the products (for example, in the case of recreational activities an attribute may be the existence of a guide). Then, from the problem of maximization of such utility function subject to linear budget restriction and non-negativity are generated the first-order conditions that dominate when a positive or zero amount of each good is consumed. Incorporating a random term (from the researcher's perspective) into the utility function makes it possible to establish probability aspects for the observed vector of the consumption set that serves as basic elements for estimating the maximum likelihood of utility function parameters.

It is important to note that in the structure of MDC prevails the structure of corner solutions. In this situation, individuals do not acquire any quantity of one or a set of available goods. Corner solutions contrast with the commonly known interior solutions, where the individual acquires positive quantities of the set of goods available. When the consumer acquires a positive quantity of up to one or two goods it is known as an extreme corner solution, however, when the consumer acquires positive quantities of more than two goods but not all of them is known as a general corner solution. The analysis of extreme corner solutions is relatively simple, however, general corner solutions are complex to analyze (Hanemann, 1984).

The main element for the utility maximization problem in the context of MDC is the use of a non-linear utility structure (increasing and continuously differentiable) with decreasing marginal utility, which presents imperfect substitution and allows the choice of multiple alternatives within a set of alternatives. According to Maler (1974) this utility function should consider the concept of weak complementarity[1], which implies that the individual does not receive utility of the attributes of a non-essential good if it does not consume it, for example in the environmental context there is weak complementarity between environmental quality and the demand for recreational activities when the demand for recreational activity is zero and there are no effects on the utility function of the individual due to improvements environmental. The main proposals for non-linear utility functions come from the Linear Expenditure System (LES) and Constant Elasticity of Substitution (CES) approaches (for more see Hanemann et al., 1978; Kim et al., 2002; von Haefen and Phaneuf, 2005 and Phaneuf and Smith, 2006). However, Bhat (2008) proposed an approach that subsumes the earlier specifications as special cases, he used a multiplicative log-extreme value error in the baseline preference for each alternative, and this was popularized as Multiple Discrete Continuous Extreme Value (MDCEV).

1.1.2 MDCEV choice model

At the beginning of its formulation, the KKT approach seemed to be attractive, however, the assumptions established by Wales and Woodland (1983) generated difficulties in the likelihood function due to multidimensional integration. Moreover, MDCEV's approach provided a closed form that was quickly used in different fields of research.

The MDCEV approach can be applied to cases to complete or incomplete demand systems. The "elementary" goods in the primary interest set may be referred to as "internal goods" (this applies to a complete or incomplete demand system). Through the Hicks approach of composite commodities, it is possible to replace "elementary" goods in each set that is not of primary interest with a single composite alternative that represents the entire set. This approach allows for the consideration of composite goods as essential goods, it is common for the Hicksian approach to use only an essential good with different inside goods.

The specifications of the utility functions for the case where only inside goods and the combination of inside goods and essential good are considered are presented below. It is important to mention that this will be presented through an Additively Separable Utility function (ASU) approach, which implies that the marginal utility of a good depends only on the consumption of that good and not on other goods, that is, with this assumption the existence of complementarity and substitution patterns is restricted.

[1]See Hanemann, 1984 and von Haefen, 2004 for a discussion.

1.1.2.1 Functional form of utility specification

1.1.2.1.1 Only inside goods

The generalization of these functional forms comes from Bhat (2008), where a Box-Cox transformation of the quantities consumed was used[2], this is

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \Psi_k \left[\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] \tag{1.1}$$

where $\boldsymbol{x}$ is the consumption vector of alternatives k ($k = 1, ..., K$), $\gamma_k > 0$, $\Psi_k > 0$, and $\alpha_k \leq 1$ for all k are required for this specification to be consistent with the properties of a utility function. In accordance with Bhat (2008) the interpretation of parameters is

- Ψ_k, represent the marginal utility of consuming alternative k at the point of zero consumption (baseline marginal utility). Since $\frac{\partial U}{\partial x_k} = \Psi_k(\frac{x_k}{\gamma_k} + 1)^{\alpha_k - 1}$, therefore, if $x_k = 0$ we have that

$$\frac{\partial U}{\partial x_k}|_{x_k=0} = \Psi_k, \tag{1.2}$$

  it is noted that the marginal utility of the good k depends only on the consumption of the same good and not of the other goods;

- γ_k, are translation parameters that allow for corner solutions (i.e. zero consumption levels for alternatives) and also influence satiation. The lower the value of γ_k, the greater the satiation effect in consuming x_k; and

- α_k, are parameters control the rate of diminishing marginal utility of additional consumption. If α_k is equal to one, then there are no satiation effects (i.e. constant marginal utility).

The satiation parameters, along with the baseline utility parameters, influence the quantity decision of individuals. However, the parameters γ_k and α_k cannot be identified separately so we have normalized the model with respect to one of these parameters, his leads to the definition of γ ($\alpha_k \to 0$, $\forall k$ to estimate γ_k parameters) and α ($\gamma_k = 1$, $\forall k$ to estimate α_k parameters) profiles, are given by

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \gamma_k \Psi_k \ln \left(\frac{x_k}{\gamma_k} + 1 \right), \tag{1.3}$$

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \frac{\Psi_k}{\alpha_k} \ln \left((x_k + 1)^{\alpha_k} - 1 \right), \tag{1.4}$$

respectively. Now, to find the optimal allocation of goods first arises the Lagrangian function for the UMP subject to a linear budget constraint, this is

[2]The Box-Cox transformation is applied over $x_k^* = (\frac{x_k}{\gamma_k} + 1)$, i.e., $(x_k^*)^{\alpha_k} = \frac{(x_k^*)^{\alpha_k} - 1}{\alpha_k}$.

$$\mathcal{L} = U(\boldsymbol{x}) + \lambda\Big(E - \sum_{k=1}^{K} p_k x_k\Big), \tag{1.5}$$

where E is the total budget and λ represents the marginal utility of income. The KKT-FOC are given by:

$$\begin{aligned}
&\Psi_k\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k = 0,\ \text{if } x_k^* > 0,\, k = 1,2,...,K,\\
&\Psi_k\left(\frac{x_k^*}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k < 0,\ \text{if } x_k^* = 0,\, k = 1,2,...,K,
\end{aligned} \tag{1.6}$$

Optimal demands satisfy equation (1.6) and budget constraint. From the budget restriction, it follows that it is necessary to estimate K-1 optimal consumption since the quantity of any good can be inferred through the consumption of all other goods. To adapt to this condition it is established that there is a good for which the individual consumes a quantity other than zero (this implies that the individual must participate at least in one of the alternative K), then, if we assume that this is true for the first good ($k = 1$), the KKT-FOC for this good can be written as

$$\lambda = \frac{\Psi_1}{p_1}\left(\frac{x_1^*}{\gamma_1}+1\right)^{\alpha_1-1}, \tag{1.7}$$

now substituting λ in the equation (1.6) for the other goods and taking logarithms, the new KKT-FOC are given by

$$\begin{aligned}
&v_k + \ln\Psi_k = v_1 + \ln\Psi_1,\ \text{if } x_k^* > 0,\, k = 1,2,...,K,\\
&v_k + \ln\Psi_k = v_1 + \ln\Psi_1,\ \text{if } x_k^* = 0,\, k = 1,2,...,K,
\end{aligned} \tag{1.8}$$

where $v_k = (\alpha_k - 1)\ln\left(\frac{x_k^*}{\gamma_k}+1\right) - \ln p_k$, for $k = 1,2,...,K$.

1.1.2.1.2 Inside goods with an essential good

If considered an essential good, then, the utility functional form needs to be modified as follows

$$U(\boldsymbol{x}) = \frac{\Psi_1}{\alpha_1}\Big((x_1+\gamma_1)^{\alpha_1}\Big) + \sum_{k=2}^{K}\frac{\gamma_k}{\alpha_k}\Psi_k\left[\left((\frac{x_k}{\gamma_k}+1)^{\alpha_k}-1\right)\right], \tag{1.9}$$

where $\gamma_1 \le 0$, $\gamma_k > 0$ for $k > 1$, note that the essential good is defined as the first alternative ($k = 1$). The magnitude of γ_1 may be interpreted as the required lower bound for consumption of the essential good. Similar to the case of the function without an essential good, the analysis will generally not be able to estimate both α_k and γ for the essential and other goods. The same procedure undertaken for the only inside goods case, this is, the KKT-FOC are the same as in (1.8), let replacing $v_j = (\alpha_k - 1)\ln(x_j^* + \gamma_k) - \ln p_k$ for essential good.

1.1.2.2 Econometric estimation

The baseline random marginal utility for each good is defined as

$$\Psi_k = \exp(\beta' z_k + \varepsilon_k),\ \forall\ k = 1, 2, ..., K, \tag{1.10}$$

where z_j is a set of individual characteristics and/or attributes of the alternative k[3] and ε_k captures the idiosyncratic (unobserved) characteristics that impact the baseline utility of good k (note that the parameterization ensures that the baseline utility is positive). Note that the term stochastic is defined for all alternatives, therefore it develops in the scenario of inside goods and at the same time is considered an essential good. Now, substituting Ψ_k (equation (1.10)) in the KKT-FOC of equation (1.8), you have to

$$\begin{aligned} V_k + \varepsilon_k &= V_1 + \varepsilon_1, \text{ if } x_k^* > 0,\ k = 2, ..., K, \\ V_k + \varepsilon_k &< V_1 + \varepsilon_1, \text{ if } x_k^* = 0,\ k = 2, ..., K, \end{aligned} \tag{1.11}$$

where $V_k = v_k + \beta' z_k$ for $k = 1, 2, ..., K$. Finally, to complete of modeling procedure, the researcher needs to define the error structure. If it is assumed that ε_k are independently distributed across all alternatives ($k = 1, 2, ..., K$) and independent of z_k, and considering a standard Extreme Value Distribution (EVD), the individual's contribution to the likelihood becomes

$$l_i = |J| \int_{\varepsilon_1=-\infty}^{\varepsilon_1=\infty} \prod_{i=2}^{K} \frac{1}{\mu} \lambda \left(\frac{V_1 - V_i + \varepsilon_1}{\mu} \right) \cdot \prod_{s=K+1}^{M} \Lambda \left(\frac{V_1 - V_s + \varepsilon_1}{\mu} \right) \frac{1}{\mu} \lambda \frac{\varepsilon_1}{\mu} d\varepsilon_1, \tag{1.12}$$

where λ and Λ denote the standard extreme value density and the cumulative distribution, respectively, M refers to the number of alternatives that are chosen and μ is scale parameter. The Jacobian becomes (Bhat (2008)):

$$|J| = \left(\prod_{i=1}^{M} c_i \right) \left(\sum_{i=1}^{M} \frac{1}{c_i} \right), \qquad \text{with} \quad c_i = \frac{1 - \alpha_i}{e_i^* + \gamma_i p_i}, \tag{1.13}$$

and after solving the integral in the likelihood function Bhat's solution is

$$l_i = \frac{1}{\mu^{M-1}} \left(\prod_{i=1}^{M} c_i \right) \left(\sum_{i=1}^{M} \frac{1}{c_i} \right) \frac{\prod_i^M \exp \frac{v_i}{\mu}}{(\sum_{k=1}^{K} \exp \frac{v_k}{\mu})^M} (M-1)!. \tag{1.14}$$

We can observe that this expression can collapses to the conditional logit model when $M = 1$, this is $l_i = \frac{\exp \frac{v_i}{\mu}}{\sum_{k=1}^{K} \exp \frac{v_i}{\mu}}$. The expression for the probability of the consumption pattern of the goods (rather

[3]for example, the context of outdoor recreation, environmental quality at different sites is an important variable that explains the behavior of site visits (Herriges and Kling, 1999 and Phaneuf and Smith, 2004.)

than the expenditure pattern) can be derived to be:

$$l_i = \frac{1}{p_1}\frac{1}{\mu^{M-1}}\left(\prod_{i=1}^{M} f_i\right)\left(\sum_{i=1}^{M}\frac{p_i}{f_i}\right)\frac{\prod_i^M \exp^{\frac{v_i}{\mu}}}{(\sum_{k=1}^{K}\exp^{\frac{v_k}{\mu}})^M}(M-1)!, \tag{1.15}$$

where $f_i = \frac{1-\alpha_i}{x_i^* + \gamma_i}$. Note that this expression is not independent of the good that is labeled as the first. That is, different probabilities of the same consumption pattern will arise depending on the good that is labeled as first. However, the term $\frac{1}{p_1}$ may be ignored from the likelihood function because it is a constant in the probability function of each individual.

Now, the expression for v_i depends on the profile to be estimated, for example for the case where only one essential good is considered, the v_i terms are given by

- $V_k = \beta' z_k + (\alpha_k - 1)\ln(x_k^* + 1) - \ln p_k \;\; \forall k > 2$ and $V_1 = (\alpha_1 - 1)\ln x_1^*$, as alpha profile, or
- $V_k = \beta' z_k - \ln(\frac{x_k^*}{\gamma_k} + 1) - \ln p_k \;\; \forall k > 2$ and $V_1 = (\alpha_1 - 1)\ln x_1^*$, as gamma profile.

As can be inferred, the use of this MDC approach allows the integration of discrete and continuous choices into a single analytical model. This is clearly a contribution to the understanding of the structure of preferences since previously there were efforts by Bockstael et al. (1987), Hausman et al. (1995) and Parsons and Kealy (1995) to integrate both decisions, they used a two-stage procedure, in a first stage they estimate a repeated logit model and then, they incorporate the variables proxy for price and quantity, which are exogenous in the estimation of demand, however, this procedure does not allow them to integrate both decisions into a single model of consumer utility.

As mentioned the MDCEV model was widely used by researchers. However, researchers are also interested in obtaining patterns of complementarity and substitution between goods, For this purpose, a constant effort has been evidenced lately by a group of researchers that extends the MDCEV approach with an ASU function towards the use of a non-additively separable utility function. This will be reviewed in detail in the next chapter.

1.1.3 Justification

The form of the utility function used in the maximization problem is important because it is the characteristics of the maximization that will allow us to deepen the interpretations and applications of the results. One of the functional forms that is commonly used corresponds to the additively separable utility (ASU) function, the use of this type of functional form for example facilitates the calculation of welfare measures. However, the ASU function does not allow capturing patterns of complementarity between products such as the non-additively separable utility (NASU) function. From the results of the maximization problem with a NASU function can be inferred relationships between different products, for example, in the context of recreational activities, it will be possible to determine whether two recreational activities, such as hunting and fishing are complementary or substitute activities.

In general, individuals make decisions of choice not only considering attributes of good to consider but in practice, they have considered attributes of a set of alternatives, of which not necessarily all should be consumed. In this sense, the study of the application of different functional forms in the problem of maximizing consumer utility with a KT approach provides important information regarding the structure of preference of individuals, since the compression of the transition of an ASU function to a NASU contributes to the exploration of the existing interrelationship within the set of alternatives when making a choice decision.

Despite the recurrent use of the ASU function, it is important to mention that it is restrictive as it reduces flexibility in terms of complementarity patterns. However, the use of a NASU function (which does allow capturing patterns of complementarity) is not easy due to the complexity of finding a likelihood function. This is worrying because the cost-benefit analysis uses the results of the use of the functional forms ASU and NASU, which are inputs for the calculation of welfare measures (Lloyd-Smith, 2018), and which are also inputs to policy design based on cost-benefit analysis (Boadway, 1974). Therefore, failure to consider complementarity patterns in the solution of the maximization problem may lead to different outcomes when they are incorporated, which in practice may lead to poor policy design.

Welfare analysis and cost-benefit analysis are conditioned to the knowledge of the structure of preference of individuals. Therefore, if the researcher does not consider that different preference structures incorporate (or not) attributes of complementarity, he may fail in the design of quantitative instruments that support policy design. For example, the effort in obtaining welfare measures depends directly on the complexity of the preference structure, therefore it is expected that results and applications are commonly presented in a context of an ASU function.

1.1.4 Objectives

1.1.4.1 Main objetive

Explore and compare the estimation of a demand system through a discrete/continuous choice model with KKT focus for a generalized corner solution with ASU and NASU functional forms.

1.1.4.2 Purpose

Evaluate the effect of the incorporation (and not) of complementarity patterns (on the preference structure) on the estimated parameters and welfare measures calculation of the discrete/continuous choice problem.

1.1.4.3 Specific objectives

1. Provide a theoretical framework for the discrete/continuous choice model through the utility maximization problem with the use of NASU functions.
2. Estimate discrete/continuous choice model using ASU functional form
3. Estimate discrete/continuous choice model using at least one NASU functional form.
4. Compare statistically the satiation and baseline utility parameters of the ASU and NASU functional forms.
5. Compare statistically the welfare measures of the ASU and NASU functional forms.

1.2 Limitations

While this paper proposes a framework for comparing the estimation of discrete-continuous choice models with ASU and NASU functional forms, the approach adopted has two main limitations. The first concerns the use of a single restriction on the consumer's problem. And the second is related to the non-consideration of time.

The use a single constraint (monetary) on the consumer problem makes it difficult to reveal socio-demographic heterogeneity in the satiety rate. In addition, the benefits of using NASU functional forms can be better exploited if one considers a restriction of total income that considers for example the budget and time of individuals (Castro et al., 2012; Astroza et al., 2017; Pellegrini et al., 2021).

The fact that the data collection process can be in several days for the same individual provides correlations and substitution between days. This implies that to incorporate the time factor it is necessary to formulate a unified model that at least considers two fundamental aspects. First, you need to enforce the day-level restrictions people face (as many restrictions as the number of days modeled). And second, consider possible interactions between the allocation of time to activities on different days, such as substitution, complementarity, and heterogeneity (Calastri et al., 2020)

1.2.1 Proposal outline

This paper is structured as follows. Chapter 2 presents the modeling of the MDC decision through the NASU approach. Chapter 3 defines the framework for calculating welfare measures in the MDC choice approach. Chapter 4 presents the results of the ASU and NASU estimates and then compares the estimated parameters and welfare measures between the different approaches. Finally, Chapter 5 discusses the main implications and contributions of the results and concludes with the limitations and future directions of research in the context of MDC choice modeling.

Chapter 2

Formulation of the Multiple discrete Continuous models with a Non-Additively Separable Utility functional forms approach

This chapter aims to define a frame of reference in incorporating NASU functional forms into the MDC choice approach. Section 2.1 provides an understanding of NASU functions and their main implications for preference analysis. Then, sections 2.2, 2.3, 2.4 and 2.5 set out the main approaches[1] that have been addressed by researchers to incorporate the NASU functions forms into the MDC choice approach, as will be shown shortly, the main challenge relates to the generalization of the Jacobian to use in the function of likelihood.

2.1 Non-Additively Separable Utility

An ASU function assumes that the marginal utility of a good is independent of the consumption of other goods. For example, if an individual participates in recreational activities such as fishing and hunting, the ASU function assumes that the marginal utility of fishing will not be affected by participation in hunting (and vice versa). This assumption is important for the following reasons.

- The marginal substitution ratio between any pair of goods i, j depends exclusively on the quanti-

[1] Each author's original notation is initially presented. It is then homogenized for comparisons.

ties consumed of the goods i, j, therefore it is independent of the other goods ($\neq i, j$). As Pollak and Wales (1995) points out, this has a direct impact on preferences. For example, consider an individual who participates in three recreational activities: hiking (x_1), bird watching (x_2), and photography (x_2). Now, consider a person who tends to hiking and bird watching, or hiking and photograph, but not just walk. This individual can choose $(20, 1, 20)$ over $(10, 10, 20)$ but also prefer $(10, 10, 1)$ over $(20, 1, 1)$. This violates the concept of additive utility because if the individual prefers $(20, 1, 20)$ over $(10, 10, 20)$, he should prefer $(20, 1, x_3)$ over $(10, 10, x_3)$ according to the utility additive. This implies that the use of an ASU function restricts the ability to incorporate flexible replacement patterns.

- The specification of a quasi-concave and increasing utility function with respect to consumption, together with the consideration of an ASU function, implies that goods cannot be inferior and cannot be complements, that is, they must be perfect substitutes (see Deaton and Muellbauer (1980))

Therefore, it is understood that the use of an ASU form is restrictive in terms of complementarity and substitution patterns in the consumption of different goods. For example, through an ASU function, it is not possible to identify whether two recreational activities such as fishing or hunting are complementary to each other.

This prompted an effort by researchers to develop theoretical frameworks that would be able to capture these patterns in the study of preferences (Song and Chintagunta, 2007; Mehta, 2007; Bunch, 2009; Lee and Allenby, 2009 and Lee et al., 2010). However, it was Vásquez and Hanemann (2008) who extended Bhat's, 2008 ASU approach and presented a quadratic version of it, through which the marginal utility of each good is allowed to depend on the level of consumption of the other goods. The approach of Vásquez and Hanemann (2008) is a NASU approach and from his work, he developed a line of research that focuses on the specification of the parameters of interaction that are incorporated in the utility function, then Bhat et al. (2015) refers to some limitations of the approach of Vásquez and Hanemann (2008), and then other extensions arise which will be discussed below.

2.2 Vásquez and Hanemman's approach

Vásquez and Hanemann (2008) extended the ASU function of Bhat (2008) allowing the marginal utility of each good to depend on the level of consumption of other goods, this is

$$U(\boldsymbol{x}) = \sum_{k=1}^{M} \frac{\gamma_k}{\alpha_k} \Psi_k \left[\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] + \frac{1}{2} \sum_{k=1}^{M} \sum_{m-1}^{M} \theta_{km} \left[\frac{\gamma_k}{\alpha_k} \left(\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \right] \left[\frac{\gamma_m}{\alpha_m} \left(\left(\frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right) \right], \quad (2.1)$$

where x_k is the quantity of good k and α_k, γ_k, θ_{km} and Ψ_k are parameters to be estimated in the model with the same assumptions, $\Psi_k \geq 0$, $\alpha_k \geq 1$ and $\gamma_k > 0$. The parameters θ_{km} allow quadratic effects as well as allow the marginal utility of good k to be dependent on the level of consumption of other goods, therefore if $\theta_{km} > 0$ the alternatives k and m are complements and if $\theta_{km} < 0$ the alternatives k and m are substitutes. Taking the derivative of (2.1) with respect to x_k

$$\frac{\partial U}{\partial x_k} = \Psi_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1} + \sum_{m=1}^{M} \theta_{km} \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1} \frac{\gamma_k}{\alpha_m} \left[\left(\frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right], \tag{2.2}$$

where (2.2) corresponds to marginal utility of good k. Now, at $x_k = 0$ we have

$$\frac{\partial U}{\partial x_k}|_{x_k=0} = \Psi_k + \sum_{m=1}^{K} \theta_{km} \frac{\gamma_m}{\alpha_m} \left[\left(\frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right], \tag{2.3}$$

in contrast to $\frac{\partial U}{\partial x_k}$ of the expression (1.1), it is now noted that the marginal utility of the good k depends on both the good k and other goods, and it is clear that Ψ_k is no longer the baseline marginal utility at the point at which good k has not been consumed. Therefore, Ψ_k represents the desirability of x_k before any consumption decision has been made.

Then, the maximization problem is

$$\max_{x_k} U(\boldsymbol{x}) = \sum_{k=1}^{M} \frac{\gamma_k}{\alpha_k} \Psi_k \left[\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] + \frac{1}{2} \sum_{k=1}^{M} \sum_{m-1}^{M} \theta_{km} \left[\frac{\gamma_k}{\alpha_k} \left(\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \right] \left[\frac{\gamma_m}{\alpha_m} \left(\left(\frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right) \right], \tag{2.4}$$

subject to

$$\sum_{k=1}^{M} p_k x_k = \sum_{k=1}^{M} e_k = E. \tag{2.5}$$

Most the literature has defined the error term inside Ψ_k either by a multiplicative approach as $\Psi_k = \exp^{\beta' z_k} \exp^{\varepsilon_k}$ or an additive approach $\Psi_k = \beta' z_k + \varepsilon_k$, and the assumptions on ε_k are either a extreme value distribution or a normal distribution. Vásquez and Hanemann (2008) showed that the multiplicative approach (in the simple case) is convenient since the FOC are reduced to an expression easy to manipulate with the logarithm operator, and these can be used directly in the likelihood function. However, in this new formulation, the multiplicative approach does not help to simplify the model.

There are two solutions to this problem. First, you can consider the additive approach, $\Psi_k = \beta' z_k + \varepsilon_k$. Second, the multiplicative approach, $\Psi_k = \exp^{\beta' z_k} \exp^{\varepsilon_k}$, can be maintained but incorporate the definition of an outside good that helps to simplify the calculation of FOC.

2.2.1 Linear stochastic structure

For this case the FOC can be expressed as

$$
\begin{aligned}
0 \leq & \frac{\beta' z_k + \varepsilon_k}{p_k}\left(\frac{e_k}{p_k\gamma_k}+1\right)^{\alpha_k-1} + \frac{1}{2}\sum_{m=1}^{M}\frac{\theta_{km}}{p_k}\left(\frac{e_k}{p_k\gamma_k}\right)^{\alpha_k-1}\frac{\gamma_m}{\alpha_m}\left(\left(\frac{e_m}{p_m\gamma_m}-1\right)^{\alpha_k-1}\right)\dots \\
& -\frac{\beta' z_k + \varepsilon_1}{p_1}\left(\frac{e_1}{p_1\gamma_1}+1\right)^{\alpha_1-1} - \frac{1}{2}\sum_{m=1}^{M}\frac{\theta_{1m}}{p_1}\left(\frac{e_1}{p_1\gamma_1}+1\right)^{\alpha_1-1}\frac{\gamma_m}{\alpha_m}\left(\left(\frac{e_m}{p_m\gamma_m}+1\right)-1\right),
\end{aligned}
\tag{2.6}
$$

where $e_k > 0$ for equality and $e_k = 0$ for inequality. The last component in expression (2.6) corresponds to the marginal utility of income λ. Now, if $a_k = (\frac{e_k}{p_k\gamma_k}+1)^{\alpha_k-1}$ and $a_1 = (\frac{e_1}{p_k\gamma_1}+1)^{\alpha_1-1}$, let the FOC can be expressed as

$$
\begin{aligned}
0 \leq & \frac{\varepsilon_k}{p_k}a_k - \frac{\varepsilon_1}{p_1}a_1 + \frac{\beta' z_k}{p_k}a_k - \frac{\beta' z_1}{p_1}a_1 + \frac{1}{2p_k}a_k\sum_{m=1}^{M}\frac{\theta_{km}\gamma_m}{\alpha_m}\left(\left(\frac{e_m}{p_m\gamma_m}+1\right)^{\alpha_m}-1\right)\dots \\
& -\frac{1}{2p_1}\sum_{m=1}^{M}\frac{\theta_{1m}\gamma_m}{\alpha_m}\left(\left(\frac{e_m}{p_m\gamma_k}+1\right)^{\alpha_m}-1\right), \\
0 \leq & \frac{\varepsilon_k}{p_k}a_k - \frac{\varepsilon_1}{p_1}a_1 + V_k - V_1 = \varepsilon_k - \frac{p_k}{p_1}\frac{a_1}{a_k}\varepsilon_1 + \frac{p_k}{a_k}V_k - \frac{p_k}{a_k}V_1, \\
\varepsilon_k \leq & \frac{p_k}{p_1}\frac{a_1}{a_k}\varepsilon_1 - \frac{p_k}{a_k}V_k + \frac{p_k}{a_k}V_1, \\
\varepsilon_k \leq & a_k^* a_1^* \varepsilon_1 - a_k^* V_k + a_k^* V_1 = V_1^* - V_k^* + \varepsilon_1^*,
\end{aligned}
\tag{2.7}
$$

this area is given by

$$
\int_{-\infty}^{V_1^*-V_k^*+\varepsilon_1} \exp^{\varepsilon_k}\exp^{\exp^{-\varepsilon_k}} d\varepsilon_k = \exp^{-\exp^{-(V_1^*-V_k^*+\varepsilon_1)}}, \tag{2.8}
$$

and the likelihood function follows a pattern similar to the case of the ASU function

$$
l_i = |J_k| \int_{\varepsilon_1=-\infty}^{\varepsilon_1=\infty} \prod_{i=2}^{K}\frac{1}{\mu}\lambda\left[\frac{V_1^*-V_k^*+\varepsilon_1^*}{\mu}\right] \prod_{s=K+1}^{M}\left[\frac{V_1^*-V_k^*+\varepsilon_1^*}{\mu}\right]]\frac{1}{\mu}\lambda\left[\frac{\varepsilon_1}{\mu}\right]d\varepsilon_1. \tag{2.9}
$$

The main problem is that there is no simple solution for the Jacobian that can be generalized to any consumption pattern and the integral in the likelihood function does not have a closed-form solution (Vásquez and Hanemann, 2008). However, the integral can be calculated by simulation or numerically, for example, if it is included a random parameter structure $\beta_n = \beta + \sigma\eta$ (β is the mean effect and σ a deviation with respect to this mean), then

$$
\begin{aligned}
L(\beta) = & \int\int_{\varepsilon_1=-\infty}^{\varepsilon_1=\infty}\prod_{i=2}^{K}\frac{1}{\mu}\exp^{-(V_1^*-V_i^*+\varepsilon_1^*)}\exp^{-\exp^{-\frac{1}{\mu}(V_1^*-V_i^*+\varepsilon_1^*)}}\dots \\
& \prod_{s=K+1}^{M}\exp^{-\exp^{-\frac{1}{\mu}(V_1^*-V_i^*+\varepsilon_1^*)}}\frac{1}{\mu}\exp^{-\varepsilon_1}\exp^{-\exp^{-\varepsilon_1}}d\varepsilon_1 f(\beta)d(\beta),
\end{aligned}
\tag{2.10}
$$

which can be calculated using simulation.

2.2.2 Outside good

The alternative to overcome the difficult manipulation of the likelihood function is to assume the existence of an outside good (Z) that does not have an error term. In this case

$$U(\boldsymbol{x}, Z) = U(\boldsymbol{x}) + \frac{1}{\rho} Z^{\rho}, \tag{2.11}$$

where U is the Quadratic Box-Cox utility function. Now, the FOC are simpler since the definition of the marginal utility of income can be expressed as

$$\frac{\partial U^*}{\partial Z} = Z^{\rho-1} - \lambda, \tag{2.12}$$

from wich $\lambda = Z^{\rho-1}$. Then, it is replaced, the FOC can be expressed as

$$\begin{aligned} 0 \leq & \frac{\exp\left(\beta' z_k + \varepsilon_k\right)}{p_k} \left(\frac{e_k}{p_k \gamma_k} + 1\right)^{\alpha_k - 1} \cdots \\ & + \frac{1}{2} \sum_{m=1}^{M} \theta_{km} \left(\left(\frac{e_k}{p_k \gamma_k} + 1\right)^{\alpha_k - 1} \right) \left(\frac{\gamma_m}{\alpha_m} \left[\left(\frac{e_m}{p_m \gamma_m} + 1\right)^{\alpha_m} - 1 \right] \right) - Z^{\rho - 1} \end{aligned} \tag{2.13}$$

and

$$\varepsilon_k \leq \beta' z_k + \ln\left(-\frac{p_k}{2} \left(\frac{e_k}{p_k \gamma_k} + 1\right)^{\alpha_k - 1} \sum_{m=1}^{M} \theta_{km} \left(\frac{\gamma_m}{\alpha_m} \left[\left(\frac{e_m}{p_m \gamma_m} + 1\right)^{\alpha_m} - 1 \right] \right) - Z^{\rho-1} \right), \tag{2.14}$$

which has the same likelihood function as the simple case except we do not have any integrals (the outside good does not have an error term). Bhat (2008) gives two arguments against the outside good formulation.

First, it is arbitrary to assume one good does not have an error term while all others are stochastic variables. Second, in his simple formulation, the likelihood function does not reduce to the simple discrete choice model when $K = 1$. However, Vásquez and Hanemann (2008) established that these arguments may be less important than the advantage of having a simple likelihood function which facilitates the estimation of this more complex model.

2.3 Bhat's approach

Bhat et al. (2015), modifies Vásquez and Hanemann (2008) functional form and proposes to delete the terms of interaction θ_{km} for all $k = m$, where three specifications are developed: Deterministic utility-random maximization (DU-RM), where the researcher knows how consumers value goods, but also recognizes that there may be a difference between the calculated optimal consumption; Random utility deterministic maximization (RU-DM), the researcher introduces stochasticity directly into the utility function to recognize that the researcher does not know all the factors that the consumer considers in his or her wine valuation pattern; and Random utility-random maximization (RU-RM), where the random utility approach is combined, as well as random maximization specifications in what can be considered the most realistic situation.

Initially, they introduce the following specification for the utility function

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \Psi_k \frac{\gamma_k}{\alpha_k}\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k}-1\right] + \frac{1}{2}\sum_{k=1}^{K}\sum_{m\neq k}\theta_{km}\left[\frac{\gamma_k}{\alpha_k}\left(\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k}-1\right)\right]\left[\frac{\gamma_m}{\alpha_m}\left(\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m}-1\right)\right], \quad (2.15)$$

Now, taking a derivative with respect to x_k and set $x_k = 0$, we have

$$\frac{\partial U}{\partial x_k} = \Psi_k\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k-1} + \sum_{m\neq k}^{M}\theta_{km}\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k-1}\frac{\gamma_k}{\alpha_m}\left[\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m}-1\right], \quad (2.16)$$

$$\frac{\partial U}{\partial x_k}|_{x_k=0} = \Psi_k + \sum_{m\neq k}\theta_{km}\frac{\gamma_m}{\alpha_m}\left[\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m}-1\right], \quad (2.17)$$

where expressions (2.16) and (2.17) represent the utility marginal of the good k and utility marginal of the good k when $x_k = 0$. This case the interpretation of Ψ_k is similarly to Vásquez and Hanemann (2008).

2.3.1 Deterministic utility-random maximization

If it is considered the NASU function of (2.15), deterministic utility form, the KKT-FOC corresponding to the utility maximization problem are

$$\begin{aligned}
&\tilde{\pi_k}\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k = 0, \quad \text{if } x_k > 0,\, k = 1,2,...,K,\\
&\tilde{\pi_k}\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k < 0, \quad \text{if } x_k = 0,\, k = 1,2,...,K,
\end{aligned} \quad (2.18)$$

where $\tilde{\pi_k}$ is the baseline marginal utility (expression (2.17)). Stochastics can now be explicitly introduced into KKT-FOC exponentially multiplicatively as follows

$$\begin{aligned}
&\tilde{\pi_k}\exp(\varepsilon_k)\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k = 0, \quad \text{if } x_k > 0,\, k = 1,2,...,K,\\
&\tilde{\pi_k}\exp(\varepsilon_k)\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k-1} - \lambda p_k < 0, \quad \text{if } x_k = 0,\, k = 1,2,...,K.
\end{aligned} \quad (2.19)$$

The probability expression collapses to the following Multiple Discrete-Continuous Extreme Value Model closed-form,

$$l_i = |J_M| \frac{1}{\mu^{M-1}} \frac{\prod_i^K \exp^{\frac{v_i}{\mu}}}{(\sum_{k=1}^{M} \exp^{\frac{v_k}{\mu}})^M} (M-1)!, \tag{2.20}$$

where this expression is similiar to Bhat (2008), however $v_k = \ln(\tilde{\pi_k}) - \ln(p_k) + (\alpha_k - 1)\ln(\frac{x_k}{\gamma_k} + 1)$ and the element of the Jacobian are given by

$$J_{ih} = \omega_{h+1} p_{h+1} \left[\frac{\theta_{1,h+1}}{\tilde{\pi_1}} - (1 - z_{ih}) \frac{\theta_{i+1,h+1}}{\tilde{\pi_{i+1}}} \right] + \omega_1 p_1^2 \frac{\theta_{1,i+1}}{\tilde{\pi_{i+1}}} + p_{h+1}(L_1 p_1 + z_{ih} L_{i+1}), \tag{2.21}$$

where $i = 1, 2, ..., M-1$, $h = 1, 2, ..., M-1$, $\omega_k = \frac{1}{p_k} \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1}$, for $k = 1, 2, .., K$, $z_{ih} = 1$ if $i = h$, $z_{ih} =$ if $i \neq h$ and $L_k = \frac{1-\alpha_k}{p_k(x_k+\gamma_k)}$. Unfortunately, there is no concise form the determinant of the Jacobian for $M > 1$. For the deterministic utility random-maximization formulation with an essential outside good, the econometrics simplify considerably.

The procedure is similar to the previous one, however now we work with expression (2.15) but for the case of an essential good (with $\beta' z_1 = 0$, $\Psi_1 = 1$, and $p_1 = 1$). then the final expression for probability in this essential good case is the same as expression in (2.20) but $v_k = \ln(\tilde{\pi_k}) - \ln(p_k) + (\alpha_k - 1)\ln\left(\frac{x_k}{\gamma_k} + 1 \right)$ with $k > 2$ and $v_1 = (\alpha_1 - 1)\ln(x_1 + \gamma_1)$.

The Jacobian elements, in this case, simplify, with $\theta_{1m} = 0$ for all $m(k \neq 1)$, the elements now are given as follows:

$$J_{ih} = \omega_{h+1} \left[-(1 - z_{ih}) \frac{\theta_{i+1,h+1}}{\tilde{\pi_{i+1}}} \right] + p_{h+1}(L_1 + z_{ih} L_{i+1}). \tag{2.22}$$

2.3.2 Random utility deterministic maximization

Considering the following NASU function for the no-essential good case

$$U(\boldsymbol{x}) = \sum_{k=1}^{K} \Psi_k \exp(\xi_k) \frac{\gamma_k}{\alpha_k} \left[\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right] + \frac{1}{2} \sum_{k=1}^{K} \sum_{m \neq k} \theta_{km} \left[\frac{\gamma_k}{\alpha_k} \left(\left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \right] \left[\frac{\gamma_m}{\alpha_m} \left(\left(\frac{x_m}{\gamma_m} + 1 \right)^{\alpha_m} - 1 \right) \right], \tag{2.23}$$

where ξ_k is IID random term with a scale parameter of σ and this term captures idiosyncratic characteristics that impact the baseline (marginal) utility of good k at the point at which no expenditure outlays have yet been made on any alternative. For expression in (2.23), the KKT-FOC are given by

$$\begin{aligned} \eta_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k = 0, \quad & \text{if } x_k > 0, k = 1, 2, ..., K, \\ \eta_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k < 0, \quad & \text{if } x_k = 0, k = 1, 2, ..., K, \end{aligned} \tag{2.24}$$

where $\eta_k = \Psi_k \exp(\xi_k) + W_k$ and $W_k = \sum_{m \neq k} = \theta_{km} \frac{\gamma_m}{\alpha_m}\left[(\frac{x_m}{\gamma_k}+1)^{\alpha_m} - 1\right]$. With definition of ω_k (expression (2.21)), let $R_k = \eta_1 \frac{\omega_1}{\omega_k}$ and $\Psi_k = \exp(\boldsymbol{\beta' z_k})$ (the first alternative be the one to which the consumer allocates some non-zero budget amount). Then, the KKT-FOC may be simplified as follows

$$\begin{aligned} \exp(\xi_k) &= \frac{R_k|\xi_1}{\exp(\boldsymbol{\beta' z_k})}, \qquad \text{if } x_k > 0,\, k = 2, ..., K, \\ \exp(\xi_k) &< \frac{R_k|\xi_1}{\exp(\boldsymbol{\beta' z_k})}, \qquad \text{if } x_k = 0,\, k = 2, ..., K. \end{aligned} \tag{2.25}$$

Then, let $\zeta_k = \exp(\xi_k)$ and assumed that $g(\cdot)$ and $G(\cdot)$ a re the standardized versions of the probability density function and standard cumulative function characterizing ζ_k, respectability. Them, the probability that the individual allocates expenditure to the first M of the K goods may be derived to be

$$l_i = \int_{\zeta_1=0}^{\zeta_1=+\infty} |J_M|\zeta_1| \left(\left[\prod_{i=2}^{M} \frac{1}{\sigma} g\left\{\frac{1}{\sigma}\frac{R_k|\xi_1}{\exp(\boldsymbol{\beta' z_k})}\right\}\right]\right)\left(\prod_{s=M+1}^{K} G\left\{\frac{1}{\sigma}\frac{R_k|\xi_1}{\exp(\boldsymbol{\beta' z_k})}\right\}\right) f(\zeta_1) d\zeta_1, \tag{2.26}$$

where $f(\cdot)$ is the density function characterizing ζ_1, and the element of Jacobian, $J_M|\xi_1$ $(i, h = 1, 2, ..., M-1)$ are given by

$$\begin{aligned} J_{ih} = \frac{1}{\exp(\boldsymbol{\beta' z_{i+1}})}\Bigg[\frac{\omega_1}{\omega_{i+1}}\Big((\eta_1|\zeta_1)(p_1^2 L_1 + p_{h+1}L_{i+1}z_{ih}) + p_{h+1}\theta_{1,h+1}\omega_{h+1}\Big) + \ldots \\ p_{h+1}\Big(p_1\theta_{1,h+1}\omega_1 - \theta_{i+1,h+1}\omega_{h+1}(1 - z_{ih})\Big)\Bigg], \end{aligned} \tag{2.27}$$

where $z_{ih} = 1$ if $i = h$, $z_{ih} = 0$ if $i \neq h$ and $L_k = \frac{1-\alpha_k}{p_k(x_k - \gamma_k)}$. The model can be estimated using maximum likelihood consumption techniques (as DU-RM formulation). If, $\eta_k > 0$ for each good k, then the marginal utility of 1 any good at any point of consumption should be positive[2].

For the essential good case the econometrics again simplify considerably, we have that $W_1 = 0$, $\boldsymbol{\beta' z_1} = 0$, $\Psi_1 = 1$, p_1 and $\eta_1 = \zeta_1 = \Psi_1 \exp(\xi_1)$. Now, the stochasticity is introduced here similar to Vásquez and Hanemann (2008) and takes the following form

$$\begin{aligned} U(\boldsymbol{x}) = \frac{1}{\alpha_1}(x_1+\gamma_1)^{\alpha_1}\eta_1 \sum_{k=2}^{K} \Psi_k \exp(\xi_k) \frac{\gamma_k}{\alpha_k}\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right] + \ldots \\ \frac{1}{2}\sum_{k=2}^{K}\sum_{m\neq k} \theta_{km}\left[\frac{\gamma_k}{\alpha_k}\left(\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right)\right]\left[\frac{\gamma_m}{\alpha_m}\left(\left(\frac{x_m}{\gamma_m}+1\right)^{\alpha_m} - 1\right)\right], \end{aligned} \tag{2.28}$$

and the probability expression take the same form as in expression (2.26) with the following modifications

$$\begin{aligned} \omega_k &= \frac{1}{p_k}\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k - 1} \qquad \text{for } k = 2, ..., K, \\ \omega_1 &= (x_1+\gamma_1)^{\alpha_1 - 1}, \end{aligned} \tag{2.29}$$

[2] For increasing utility functions

and then the elements again can be computed in closed form and are as follows ($i, h = 1, 2, ..., M - 1$):

$$J_{ih} = \frac{1}{\exp\left(\boldsymbol{\beta}' \boldsymbol{z_{i+1}}\right)} \left[\frac{\omega_1}{\omega_{i+1}} \left\{ (\eta_1|\zeta_1)(L_1 + p_{h+1} L_{i+1} z_{ih}) - p_{h+1} \theta_{i+1,h+1} \omega_{h+1} (1 - z_{ih}) \right\} \right]. \quad (2.30)$$

2.3.3 Random utility random maximization

From (2.23) for the case with no essential good but now add stochasticity originating from consumer mistakes in the optimizing process, the KKT-FOC corresponding to the utility maximization problem are

$$\begin{aligned} \eta_k \exp \varepsilon_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k = 0, \quad & \text{if } x_k > 0, k = 1, 2, ..., K, \\ \eta_k \exp \varepsilon_k \left(\frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k < 0, \quad & \text{if } x_k = 0, k = 1, 2, ..., K, \end{aligned} \quad (2.31)$$

where η_k has the same definition as above, and the ε_k are independent and identically extreme value distributed[3]. Let $Var(\varepsilon_k) + Var(\xi_k) = \frac{\pi^2 \sigma^2}{6}$ for $k = 1, 2, ..., K$. In the RU-RM specification it is assumed that the ξ_k terms are normally distributed, this is particularly convenient when one wants to accommodate a flexible error covariance structure through a multivariate normal-distributed coefficient vector β and/ or account for covariance in utilities across alternatives through the appropriate random multivariate specification for the ξ_k terms. To developed the probability function for consumptions, let $Var(\varepsilon_k) = \frac{\mu^2(\pi^2\sigma^2)}{6}$ and $Var(\xi_k) = \frac{(1-\mu^2)(\pi^2\sigma^2)}{6}$ for $k = 1, 2, ..., K$, where μ is a parameter to be estimated ($0 \leq \mu \leq 1$), the if $\mu \to 0$, and when there is no covariance among the ξ_k terms across alternatives, the RU-RM specification approaches the RU-DM specification. However, if $\mu \to 1$, the RU-RM specification approaches the DU-RM specification.Thus, the parameter μ determines the extent of the mix of the RU-DM and DU-RM decision postulates leading up to the observed behavior of consumers. One can impose the constraint that $0 \leq \mu \leq 1$ through the use of a logistic transforms $\mu = \frac{1}{1 - \exp(-\mu^*)}$ and estimate the parameter μ^*.

The probability expression for consumptions in the RU-RM model specification takes the following mixed MDCEV form

$$l_i = \int_{\xi=-\infty}^{\infty} \left[|J_M|\xi| \frac{1}{(\mu\sigma)^{M-1}} \left[\frac{\prod_{i=1}^{M} \exp\left(\frac{V_i}{\mu\sigma}\right)|\xi_i}{\sum_{k=1}^{K} \exp\left(\frac{V_i}{\mu\sigma}\right)|\xi_i} \right] (M-1)| \right] dF(\xi), \quad (2.32)$$

where $V_k = \ln(\eta_k) - \ln(p_k) + (\alpha_k - 1)\ln(\frac{x_k}{\gamma_k} + 1)$,$\eta_k = \Psi_k \exp(\xi_k) + W_k$, W_k is defined as earlier, and F is the multivariate normal distribution of the random element vector $\xi = (x_1, x_2, ..., x_K)$[4]. Then, the elements of the Jacobian are given by

[3]Recall that the ξ_k terms represent stochasticity due to the analyst's inability to capture consumer preferences. ε_K terms represent stochasticity due to consumer errors in utility maximization.

[4]Each of whose elements has a variance of $\frac{(1-\mu^2)(\pi^2\sigma^2)}{6}$.

$$J_{ih} = \omega_{h+1} p_{h+1} \left[\frac{\theta_{1,h+1}}{(\eta_1|\xi_1)} - (1 - z_{ih}) \frac{\theta_{i+1,h+1}}{(\eta_{i+1}|\xi_{i+1})} \right] + \omega_1 p_1^2 \frac{\theta_{i+1,h+1}}{(\eta_{i+1}|\xi_{i+1})} + p_{h+1}[L_1 z_{ih} L_{i+1}]. \quad (2.33)$$

When there is an essential essential good, the probability expression remains the same, but with $V_k = \ln(V_k) - \ln(p_k) + (\alpha_k - 1)\ln(\frac{x_k}{\gamma_k} + 1)$ for $k > 2$, $V_1 = (\alpha_1 - 1)\ln(x_1 + \gamma_1)$, $\theta_{1m} = 0$ for all $m(m \neq 1)$, $W_1 = 0$, $\boldsymbol{\beta' z_1} = 0$, $\Psi_1 = 1$, $p_1 = 1$, and $\eta_1 = \exp(\xi_1)$. Then the Jacobian elements in this case are given as follows

$$J_{ih} = \omega_{h+1} \left[-(1 - z_{ih}) \frac{\theta_{i+1,h+1}}{(\eta_{i+1}|\xi_{i+1})} \right] + p_{h+1}[L_1 + z_{ih} L_{i+1}]. \quad (2.34)$$

Similar to the earlier two formulations, $\eta_k > 0$ for $k = 1, 2, ..., K$ is required for that the marginal utility of consumption for any alternative always is positive.

2.4 Musalem's approach

Musalem et al. (2013) indicates that both the contributions of Vásquez and Hanemann (2008) and Bhat et al. (2015) have a problem, which is that the number of parameters to be estimated grows rapidly with a number of alternatives. Musalem et al. (2013) model the agent's utility as follows

$$U(\overrightarrow{q}) = \sum_{s=1}^{S} g_s \left(\sum_{n \in N} u_{ns}(q_{ns}) \right) + u_0(q_0), \quad (2.35)$$

where $u_{ns(q_{ns})}$ is the utility contribution of consuming good $n \in N_s$, which should be increasing concave and equal to zero when $q_{ns} = 0$, g_s is the total utility derived from all goods in subset s, wich should be increasing concave and equal to zero when $\sum_{n \in N_s} = 0$, y $u_0(q_0)$ is the utility derived from consuming the outside good, which should be increasing concave and equal to zero when $q_0 = 0$. Then, the utility contribution of good n to subset s, $u_{ns}(q_{ns})$, is

$$u_{ns}(q_{ns}) = \frac{\delta_{ns}}{\rho}(-1 + (1 + q_{ns})^{\rho}), \quad (2.36)$$

where $\delta_{ns} > 0$ representing preference for good n, ρ is a diminishing returns (satiation), this parameter that allows the matginal utility of good n to change with q_{ns}. They write sub-set specific utility in a similar mode

$$g_s \left(\sum_{n \in N_s} u_{ns}(q_{ns}) \right) = \frac{\kappa_s}{\gamma} \left(-1 + \left\{ 1 + \sum_{n \in N_s} u_{ns}(q_{ns}) \right\}^{\gamma} \right), \quad (2.37)$$

where $\kappa_s > 0$ represents a preference for any positive consumption in s and γ is a diminishing returns parameter that allows the marginal utility of subset s to change with consumption of any of its elements. iF $\gamma = 1$, then (2.37) collapses to an additively separable formulation which is to Bhat (2008), however γ is a additional parameter to relax additive separability.They asumme that,

$$u_0(q_0) = \frac{\delta_0}{\rho} q_0^{\rho}, \tag{2.38}$$

where δ_0 controls the magnitude of the outside good contribution to total utility. Then, given these definitions ((2.37) and (2.38)) we have that

$$\frac{\partial U(\overrightarrow{q}, \overrightarrow{\theta})}{\partial q_n} = \left(1 + g_s \left\{ \sum_{n \in N_s} u_{ns}(q_{ns}) \right\}\right)^{\gamma - 1} \kappa_s \delta_{ns} (1 + q_{nst})^{\rho - 1}, \tag{2.39}$$

where for any pair of goods n and n' belonging to the same subset

$$\frac{\frac{\partial U(\overrightarrow{q}, \overrightarrow{\theta})}{\partial q_n}}{\frac{\partial U(\overrightarrow{q}, \overrightarrow{\theta})}{\partial q_{n'}}} = \frac{\delta_{ns}(1 + q_{ns})^{\rho - 1}}{\delta_{n's}(1 + q_{n's})^{\rho - 1}}, \tag{2.40}$$

where the ratio depends on the parameters of alternatives n and n' and ρ, however not on any other δ_{js} parameters for $j \notin \{n, n'\}$.

Regarding the interpretation of parameters: κ_s modifies the marginal utility of consuming any alternative within subset s; δ_{ns} directly affects the marginal utility of consuming good n within a subset s; ρ and γ determine the optimal distribution of spending across and within subsets, on the one hand, ρ governs the rate at which subset utility decreases with spending on any single good within a subset and on the other hand, γ is the rate at which total utility varies across goods within a subset. The combinations of values of the and parameters and the optimal strategies employed by the agent are described in the table 2.1.

Table 2.1: Different budget allocation strategies as a function of ρ and γ

	ρ low	ρ high
γ high	many alternatives in few subsets	few alternatives in few subsets
γ low	many alternatives in many subsets	few alternatives in many subsets

source: Musalem et al. (2013)

For parameterization Musalem et al. (2013) assumes that

$$\delta_{inst} = \exp(\beta_i' \overrightarrow{x}_{nst} + \epsilon_{inst}), \tag{2.41}$$

where $\overrightarrow{x}_{nst}$ is a vector of observed good characteristics (including a constant); β_i is the agent's vector of preference weights for those characteristics; and ϵ_{inst} represents unobserved characteristics that modify the utility contribution of the good n belonging to subset s for agent i during period t[5]. For the outside good they assume that $\delta_{i0t} = \exp(\epsilon_{i0t})$. For parameterize subset utility it is assumed

[5] The exponential function ensures inst¿ 0 else product n is not a "good".

$$\kappa_{ist} = \exp(v_i' \overrightarrow{w}_{st} + \varepsilon_{ist}), \tag{2.42}$$

where $\overrightarrow{w}_{st}$ is a vector of observed subset characteristics on occasion t; v_i is the agent's vector of preference weights for those characteristics; and ε_{ist} denotes unobserved subset characteristics that modify the utility contribution of all good belonging to subset "s for agent i in the period t. Then, from (2.41) and (2.42) the equation (2.35) can be expressed as

$$U(\overrightarrow{q}_{it}) = \sum_{s=1}^{S} \frac{\exp(\beta_i' \overrightarrow{x}_{nst} + \epsilon_{inst})}{\gamma_i} \left(-1 + \left\{ \sum_{n \in N_{st}} \frac{\exp(\beta_i' \overrightarrow{x}_{nst} + \epsilon_{inst}}{\rho_i} (-1 + (1 + q_{inst})^{\rho_i}) \right\}^{\gamma_i} \right) + \ldots \frac{\epsilon_{i0t}}{\rho_i} q_{i0t}^{\rho_i \gamma_i}, \tag{2.43}$$

where ϵ_{inst}, ϵ_{i0t} and ε_{ist} are know to agent i at the time t but not to be econometrician. Thus, the errors represent researcher uncertainty about preferences. ϵ_{inst} and ϵ_{i0t} are assumed to be independently distributed extreme value with zero mode and scale σ_ϵ and ε_{ist} are assumed to be independently normally distributed with zero mean and variance σ_ε^2.

Then, S^+ and S^0 are the collection of subsets with positive consumption and the set of subsets with zero demand, respectively. For any subset $s \in S^+$, let N_s^+ denote the set goods with positive consumption in the subset and let its complement be $N_s^0 = N_s / N_s^+$. Define m_s as the smallest index N_s^+[6]. For any $n \in N_s^0$ the ratio KKT-FOC for goods m_s and $n \in N_s^0$ can be written as

$$\frac{\delta_{ns}(1 + q_{ns})^{\rho - 1}}{\delta_{m_s s}(1 + q_{m_s s})^{\rho - 1}} < \frac{p_{ns}}{p_{m_s s}}, \tag{2.44}$$

taking logs and substituting in for δ_{ns} and $\delta_{m_s s}$,

$$\epsilon_{ns} < V_{m_s s} + \epsilon_{m_s s} - V_{ns}, \tag{2.45}$$

for all $n \in N_s^0$ and $V_{ns} = \beta \overrightarrow{x}_{ns} + (\rho - 1)\ln(1 + q_{ns}) - \ln(p_{ns})$. Then, for $n \in N_s^+$, the log ratio of KKT-FOC of n and m_s implies

$$\epsilon_{ns} = V_{m_s s} + \epsilon_{m_s s} - V_{ns}. \tag{2.46}$$

for all $n \in N_s^+$ with $n \neq m_s$. And ε_s is obtained from the log ratio of KKT-FOC for goods m_s and the outside good

$$\epsilon_s = -v' w_s - (\gamma - 1)\ln\left(1 + \sum_{n \in N_s} u_{ns}(q_{ns})\right) - V_{m_s s} - \epsilon_{m_s s} + (\rho\gamma - 1)\ln(q_0) + \epsilon_0. \tag{2.47}$$

Then, the likelihood of the subset is

[6] For example, if the agent purchases 0 units of good 1, 6 unit of good 2 and 4 units of good 3 in a three-good subset s. , then $N_s = \{1, 2, 3\}$, $N_s^+ = \{2, 3\}$, $N_s^0 = \{1\}$ and $m_s = 2$.

$$\mathcal{L}^{+}(\{\overrightarrow{q}_s\}_s \in S^{+}|\{\epsilon_{m_s s}\}_{s\in S^{+}}, \epsilon_0)$$
$$= \Big[\prod_{s\in S^{+}}\prod_{n\in N_s^0} F(\epsilon_{ns} < V_{m_s s} + \epsilon_{m_s s} - V_{ns})\Big]$$
$$\cdot\Big[\prod_{n\in S^{+}}\prod_{n\in N_s^{+}, n>m} f(\epsilon_{ns} = V_{m_s s} + \epsilon_{m_s s} - V_{ns})\Big] \tag{2.48}$$
$$\cdot\Big[\prod_{s\in S^{+}} f(\epsilon_s = -v' w_s - (\gamma-1)\ln(A_s(\overrightarrow{q}_s)) - V_{ms} - \ldots$$
$$- \epsilon_{m_s s} + (\rho-1)\ln(q_0) + \epsilon_0)\Big] \cdot |J|,$$

where $f(\cdot)$ is the probability density function and $F(\cdot)$ is the cumulative distribution function of a random variable; $A_s(\overrightarrow{q}_s) = 1 + \sum_{n\in N} u(q_{ns})$; J is the Jacobian of the transformation of all positive quantities into the unobserved good and subset characteristics; and $|J|$ denotes the absolute value of the determinant of the Jacobian. The Jacobian expression given by

$$\frac{\partial \epsilon_s}{\partial q_{n's'}} = \begin{cases} -\frac{(\rho-1)p_{n's'}}{q_0} & \text{if } s \neq s', \\ -\frac{(\rho-1)p_{n's'}}{q_0} - \frac{(\rho-1)}{1+q_{n's'}} - \frac{\gamma-1}{A_s(\overrightarrow{q_s})}\Big(\exp^{\beta' x_{n's'}+\epsilon_{n's'}}(1+q_{n's'})^{\rho-1} \\ +\frac{(\rho-1)}{1+q_{n's'}}\sum_{n\in N_s, n>m_s} u(q_{ns})\Big) & \text{if } s = s' \text{ and } n' = m_s, \\ -\frac{(\rho\gamma-1)p_{n's'}}{q_0} - \frac{\gamma-1}{A_c(\overrightarrow{q_s})}\Big(\exp^{\beta' x_{n's'}+\epsilon_{n's'}}(1+q_{n's'})^{\rho-1} \\ -\frac{(\rho-1)}{1+q_{n's'}} u(q_{n's'})\Big) & \text{if } s = s' \text{ and } n' > m_s. \end{cases}$$

(2.49)

Now, consider a subset $s \in S^0$. The log ratio between the KKT-FOC for each good $n \in s$ and the outside good implies

$$\epsilon_{ns} < (\rho\gamma - 1)\ln(q_0) + \epsilon_0 - v' w_s - \varepsilon_s - V_{ns}, \tag{2.50}$$

if and only if $q_{ns} = 0$. Conditioning on $\{\varepsilon : s \in S^0\}$ and ϵ, the likelihood of observing no spending in these subsets is given by

$$\mathcal{L}^{0}(\{\overrightarrow{q}_s\}_s \in S^0|\{\epsilon_s\}_{s\in S^0}, \epsilon_0) = \prod_{s\in S^0}\prod_{n\in N_s} \mathrm{p}(\epsilon_{ns} < (\rho\gamma-1)\ln(q_0) + \epsilon_0 - v' w_s - \varepsilon_s - V_{ns}). \tag{2.51}$$

Finally, the full likelihood contribution of one agent on one choice occasion is

$$\mathcal{L}(\overrightarrow{q}|\{\epsilon_{m_s s}\}_{s\in S^+},\{\epsilon_s\}_{s\in S^0},\epsilon_0) = \mathcal{L}^+(\{\overrightarrow{q}_s\}_{s\in S^+}|\{\epsilon_{m_s s}\}_{s\in S^+},\epsilon_0)\mathcal{L}^0(\{\overrightarrow{q}_s\}_{s\in S^0}|\{\epsilon_s\}_{s\in S^0},\epsilon_0), \tag{2.52}$$

which depend on the values of unobserved good and subset characteristics, so the unconditional likelihood is obtained by integration over their distributions.

2.5 Pellegrini's approach

Pellegrini et al. (2019) extend the model at Bhat et al. (2015) through the incorporation of a new interaction term into the utility function[7]. Propose two functional forms, first

$$U(x) = \sum_{k=1}^{K} \psi_k \gamma_k \ln\left(\frac{x_k}{\gamma_k}+1\right) + \frac{1}{2}\sum_{k=1}^{K}\sum_{m\neq k}\theta_{km}\ln(x_k+1)\ln(x_m+1). \tag{2.53}$$

This functional form is similar to that presented by Deaton and Muellbauer (1980), however, this case allows corner solutions. And second,

$$U(x) = \sum_{k=1}^{K} \psi_k \gamma_k \ln\left(\frac{x_k}{\gamma_k}+1\right) + \frac{1}{2}\sum_{k=1}^{K}\sum_{m\neq k}\theta_{km}\ln(x_k\cdot x_m+1), \tag{2.54}$$

where, complementary and substitution patterns are shaped through a product of pair goods within a single logarithm. The KKT-FOC are given by

$$\begin{aligned}
&\left(\frac{x_k^*}{\gamma_k+1}\right)^{-1}\psi_k + \sum_{m\neq k}^{K}\frac{\ln(x_m+1)\theta_{mk}}{x_k+1} - \lambda p_k = 0,\ \text{if } x_k^*>0,\ k=1,...,K,\\
&\left(\frac{x_k^*}{\gamma_k+1}\right)^{-1}\psi_k + \sum_{m\neq k}^{K}\frac{\ln(x_m+1)\theta_{mk}}{x_k+1} - \lambda p_k < 0,\ \text{if } x_k^*=0,\ k=1,...,K,
\end{aligned} \tag{2.55}$$

for the first functional form, and

$$\begin{aligned}
&\left(\frac{x_k^*}{\gamma_k+1}\right)^{-1}\psi_k + \sum_{m\neq k}^{K} x_m\theta_{mk} - \lambda p_k = 0,\ \text{if } x_k^*>0,\ k=1,...,K,\\
&\left(\frac{x_k^*}{\gamma_k+1}\right)^{-1}\psi_k + \sum_{m\neq k}^{K} x_m\theta_{mk} - \lambda p_k < 0,\ \text{if } x_k^*=0,\ k=1,...,K,
\end{aligned} \tag{2.56}$$

for the second functional form. The estimation strategies used by Musalem et al. (2013) are similar to those proposed by Bhat et al. (2015).

[7]The definition of the parameters is the same as in Vásquez and Hanemann (2008) and Bhat et al. (2015).

Chapter 3

Welfare measures

This chapter presents the general approach to the calculation of welfare measures in the MDC approach. It begins with a review of the first contributions. Then, the approach commonly used for ASU functions is presented. And finally, an alternative procedure is presented that allows for estimating welfare measures for NASU functions.

The calculation of welfare measures for unified demand systems was explored by Von Haefen et al. (2004), who allowed the prevalence of corner and interior solutions and present his proposal as an alternative to the theoretical framework of the Random Utility Model, which was widely used due to computational limitations that existed to work with demand systems.

While working with demand systems involves essentially two problems (choice of parsimonious utility function and specification of unobserved heterogeneity), if these are handled correctly, KT's approach to demand systems is potentially appropriate. However, the calculation of welfare measures for welfare systems presents additional difficulties.

The calculation of welfare measures comes from a restricted optimization problem. The compensatory variation (CV) for a change from (p^0, q^0) to (p^1, q^1) can be implicitly expressed using the indirect utility function through

$$V(p^1, q^1, y - CV, \Theta, \varepsilon) = V(p^0, q^0, y, \Theta, \varepsilon), \tag{3.1}$$

where y is income and Θ is a vector of parameters (Ψ_k, γ_k and β to ASU function and Ψ_k, γ_k, β and θ_{jk} to NASU function). This is not easy to fix because there is no closed solution for CV and iterative numeric bisection algorithms are required to solve, against this Phaneuf et al. (2000) developed a strategy to analytically find each of the 2^K possible indirect utility functions and determine what is the maximum, however, this proposal seems to work well when the number of alternatives is small, it becomes more complicated. To address this problem, researchers have designed different routines that depend on some properties of utility functions.

3.1 ASU functions

Von Haefen et al. (2004) developed an iterative routine that uses KKT conditions and the separable additivity property to find optimal consumption levels and find a solution for the CV, However, the numeric bisection routine is complex to deal with due to the use of the implicit definition of CV (see equation (3.1)), the theoretical framework of this routine is that it is only necessary to find the level of consumption of an external good in order to find the levels of consumption of all other goods, therefore to separate that outside good are provided with separable additivity of the utility function. von Haefen (2007) proposes the use of the explicit definition of CV to make the Von Haefen et al. (2004) routine[1] more efficient, where it finds optimal consumption levels that minimize the expenditure function that allow to satisfy the following explicit expression of CV

$$CV = y - e(p^1, q^1, U^0, \Theta, \varepsilon), \tag{3.2}$$

where $e(\dots)$ is the function of expenditure and $U^0 = V(p^0, q^0, y, \Theta, \varepsilon)$ is the initial utility level. Lloyd-Smith (2018) modifies the algorithm proposed by von Haefen (2007) and makes it more efficient for certain scenarios, however both the routines of von Haefen (2007) and Lloyd-Smith (2018) (independent of computational efficiency) present a great limitation in common, both are based on separable additivity of utility functions, this implies these routines do not work under the assumption of separable non-additivity.

3.2 NASU functions

Considering functional forms other than ASU adds an additional problem to the calculation of welfare measures. This is mainly because the algorithm proposed by Von Haefen et al. (2004) (seminal for von Haefen (2007) and Lloyd-Smith (2018)) separates the external good from the others, however, in a

[1]To find the details of the bisection algorithm check von Haefen (2007).

NASU function this is not possible because the level of consumption of the external good depends on the other goods.

Vásquez and Hanemann (2008) stress that the KKT conditions of the NASU problem prevent the separation technique from the external good applied by Von Haefen et al. (2004), therefore the algorithm is not adequate. This is why they propose a restricted optimization routine to find the optimal consumption levels (outside good and others) that minimize the spending function restricted to non-negativity conditions and the initial utility level (U^0). With the consumption levels found, the level of expenditure necessary to reach (U^0) is calculated and thus obtains the measure of welfare expressed in equation (3.2).

This paper works with the approach of Vásquez and Hanemann (2008), since this approach can be applied in both ASU and NASU cases.

Chapter 4

Estimation and Results

This chapter presents estimates for different approaches in modeling MDC choice. First, it describes the data set used and presents some considerations. Second, estimation considerations are set out for both the ASU and NASU approaches. And third, the parameters and measures of welfare estimated through the functional forms considered are presented and compared.

4.1 Data description

The data used comes from the Canadian Nature Survey and was obtained through (Lloyd-Smith, 2020). Options are presented for the number of days dedicated to recreation in 17 outdoor activities for 500 individuals. The data includes 17 recreation alternatives and 500 individuals are surveyed to find out how many days they are willing to spend on each of the alternatives. The data includes information on three individual characteristics: university (a dummy variable if the person has completed a university degree), age index (a person's age divided by the average age in the sample), and urban (a dummy variable if a person lives in an urban area).

However, this document considers one subset[1] of alternatives: Nature-based Recreation (NBR), which considers recreational activities such as: hiking, beach, camping, cycling, ski cross, ski down and golf. This consideration is due to the fact that, as demonstrated in the literature, the number of parameters to be estimated grows rapidly when considering a high number of alternatives and even more in the NASU approach. For the final estimate, only individuals are considered to consume at least one recreational alternative (according to Bhat et al. (2015)). This reduces the NBR sample from 500 individuals to 330, this is because the alternative considered as essential good is Hiking, which presents a positive consumption in 330 individuals of the sample.

Table 4.1 below shows how many alternatives individuals choose from the sample, only 65 individu-

[1]The different types of sets of recreational alternatives are presented in the Canadian National Survey.

als choose only one alternative, therefore the other individuals (80.30%) choose at least two alternatives. In table 4.2 shows the number of individuals who consume and do not consume each of the alternatives, and the average consumption and prices of each of the alternatives

Table 4.1: Choice of individuals

Alternatives chosen	Number of individuals	% of the sample
1	65	19.70%
2	72	21.82%
3	72	21.82%
4	52	15.76%
5	41	12.42%
6	19	5.76%
7	9	2.73%

Note: note that the sample does not have individuals choosing zero alternatives, this is because all individuals choose at least one alternative, which corresponds to the essential good.

Table 4.2: What goods do individuals consume?: NBR case

Alternative (good)	Not consume	Consume	Average consumption	Average price
hiking	0	330	59.879	32.072
beach	138	192	8.727	55.106
camping	224	106	3.094	62.350
cycling	201	129	12.382	43.366
golf	233	97	5.303	130.011
ski_cross	240	90	3.545	32.316
ski_down	259	71	2.100	153.620

Note: the second and third column (they consume and do not consume) shows the number of individuals who consume and do not consume each of the goods. So the number of individuals who consume the good x plus the number of individuals who do not consume the good x add up to the number of total individuals. The fourth column shows the average consumption of each good.

4.2 Estimation considerations

4.2.1 Form ASU

From the functional form in (1.9) (for the case of an essential good) and using the gamma profile with $\alpha_k = 0 \ \forall k$, an optimization algorithm is constructed to find the β and γ vectors that maximize the maximum likelihood function for the expression in (1.15). Two variables specific to individuals are considered: age and urban index (with a constant). According Bhat (2008), the following identification and stability conditions for estimation are considered:

1. The α parameters are set to 0 for all $k = 1, ..., K$ or $\alpha_1 = \alpha_2 = ... = \alpha_k = 0$ (i.e. essential and other alternatives).

2. Parameter μ is set at 1.

3. The price of the essential good is set at 1, i.e., $p_1 = 1$.

4. Ψ_1 not include any observed explanatory variables as the coefficients of all explanatory variables for this alternative are normalized to zero.

5. To satisfy the conditions of $\gamma_k > 0$ and $\Psi_k > 0$, it is parameterized as $\gamma_k = \exp(\gamma_k^*)$ and $\Psi_k = \exp(\beta' z_k)$.

6. The essential good is defined as the first alternative of the set of alternatives, for this case: Hiking.

Through these considerations it will be possible to estimate $K-1$ vectors of explanatory variables coefficients, $K-1$ γ parameters and $K-1$ Ψ parameters. Castro et al. (2012) found that similar constraints for the γ-profile provide stable estimates.

4.2.2 Form NASU

For this case we will work with the gamma profile of the general case presented by Bhat et al. (2015), that is.

$$U(x) = \sum_{k=1}^{K} \Psi_k \gamma_k \ln\left(\frac{x_k}{\gamma_k} + 1\right) + \frac{1}{2} \sum_{k=1}^{K} \sum_{m \neq k}^{K} \theta_{km} \gamma_k \gamma_m \ln\left(\frac{x_k}{\gamma_k} + 1\right) \ln\left(\frac{x_m}{\gamma_m} + 1\right), \tag{4.1}$$

this decision implies that the BMU level will be

$$\pi = \frac{\partial U(x)}{\partial x_k} |_{x_k=0} = \Psi_k + \sum_{m \neq k}^{K} \theta_{km} \gamma_m \ln\left(\frac{x_m}{\gamma_m} + 1\right). \tag{4.2}$$

Then the likelihood function corresponds to the expression in (2.20). The general estimation considerations are similar to those of the ASU case, however, new conditions are incorporated due to consideration of the interaction parameters. Regarding the terms of interaction, the interaction matrix must initially be defined as

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_{11} & \theta_{12} & \dots & \theta_{1k} \\ \theta_{21} & \theta_{22} & \dots & \theta_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{j1} & \theta_{j2} & \dots & \theta_{jk} \end{bmatrix} \tag{4.3}$$

then, considering that $\theta_{ij} = 0 \, \forall \, i = j$ and $\theta_{1k} = 0 \, \forall \, k \neq 1$ (Bhat et al., 2015) the interaction matrix, for the example of 7 alternatives[2], is defined as

$$\boldsymbol{\theta} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta_{2,3} & \theta_{2,4} & \theta_{2,5} & \theta_{2,6} & \theta_{2,7} \\ 0 & \theta_{3,2} & 0 & \theta_{3,4} & \theta_{3,5} & \theta_{3,6} & \theta_{3,7} \\ 0 & \theta_{4,2} & \theta_{4,3} & 0 & \theta_{4,5} & \theta_{4,6} & \theta_{4,7} \\ 0 & \theta_{5,2} & \theta_{5,3} & \theta_{5,4} & 0 & \theta_{5,6} & \theta_{5,7} \\ 0 & \theta_{6,2} & \theta_{6,3} & \theta_{6,4} & \theta_{6,5} & 0 & \theta_{6,7} \\ 0 & \theta_{7,2} & \theta_{7,2} & \theta_{7,4} & \theta_{7,5} & \theta_{7,6} & 0 \end{bmatrix}. \tag{4.4}$$

Therefore, the interaction parameters to estimate correspond to $\theta_{jk} \; \forall \; j, k = 2, .., K$ excluding $j = k$ terms.

4.3 Estimation results

This paper considers one subset[3] of alternatives: Nature-based Recreation (NBR), which considers recreational activities such as: hiking, beach, camping, cycling, ski cross, ski down and golf. This consideration is due to the fact that, as demonstrated in the literature, the number of parameters to be estimated grows rapidly when considering a high number of alternatives and even more in the NASU approach. For the final estimate, only individuals are considered to consume at least one recreational alternative (according to Bhat et al. (2015)). This reduces the NBR sample from 500 individuals to 330, this is because the alternative considered as essential good is Hiking, which presents a positive consumption in 330 individuals of the sample. The following is the descriptive statistics of the group of individuals to consider.

4.3.1 Satiation parameters

Parameters γ in addition to allowing corner solutions (Vásquez and Hanemann, 2008 and Bhat et al., 2015) also serve as parameters of satiety. That is, the higher the value of γ, the lower the satiety effect on the consumption of the x alternative. Therefore, to evaluate if there are differences between these estimated parameters through the different approaches we can consider the following hypothesis contrast

[2] The essential good corresponds to the alternative 1, $j = 1$.

[3] The different types of sets of recreational alternatives are presented in the Canadian National Survey.

$$H_0 : \gamma_{j,ASU} = \gamma_{j,NASU},$$
$$H_1 : \gamma_{j,ASU} \neq \gamma_{j,NASU}, \tag{4.5}$$

where γ_{ji} corresponds to the estimated satiety parameters for the j alternative through the ASU or NASU approach. Table 4.3 presents the estimation results of the γ parameters, note that although the parameters through the NASU approach are minor, there is no evidence of a significant difference between the two approaches.

Table 4.3: $\hat{\gamma}_k$ parameters

Alt.	ASU		NASU	
	est.	s.e.	est.	s.e.
Beach	4.88	0.74	4.49	0.53
Camping	4.32	0.72	4.09	0.64
Cycling	14.03	2.66	13.81	1.31
Golf	21.9	7.39	10.14	1.91
Ski cross	5.63	1.07	5.56	1.05
Ski down	5.97	1.42	5.08	1.16

Note: the $s.e.(\hat{\gamma})$ was calculated through the delta method.

4.3.2 Baseline marginal utility parameters

As presented in previous sections, the term BMU is not similar for the ASU (equation (1.2)) and NASU (equation (2.17)) approaches, as the latter contains the term interaction between alternatives. Now, it is initially possible to review differences through the following hypothesis contrast

$$H_0 : BMU_{j,ASU} = BMU_{j,NASU},$$
$$H_1 : BMU_{j,ASU} \neq BMU_{j,NASU}, \tag{4.6}$$

where BMU_j corresponds to the Baseline Marginal Utility of the alternative j through the ASU or NASU approach. Table 4.4 shows that for this case there are no significant differences between the BMU estimated for both approaches, this may be because the estimated interaction parameters are small values, so eventually, if these were "larger" could show significant differences.

Table 4.4: BMU parameters

Alt.	ASU		NASU	
	est.	s.e.	est.	s.e.
Beach	3.43	0.57	3.74	0.44
Camping	1.19	0.23	1.06	0.27
Cycling	1.5	0.27	1.46	0.24
Golf	2.6	0.51	2.89	0.55
Ski cross	0.8	0.15	0.81	0.28
Ski down	2.47	0.52	2.67	0.56

Note: the $s.e.(BMU)$ was calculated through the delta method. The consumption values of the other alternatives are set at mean.

4.3.3 Interactions parameters

Table 4.5 presents the results for estimates of interaction parameters. It is important to remember that understanding these parameters will allow inferring whether the alternatives are complementary or substitutes, as observed, only Camping-Ski down, Golf-Ski down and Cycling-Ski down have a parameter $\theta_{ij} > 0$, that is, they have complementarity relationshiop. This implies that all relationships between the other pairs of alternatives are substitution.

Table 4.5: $\hat{\theta}_k$ parameters

Alt.	ASU		NASU	
	est.	s.e.	est.	s.e.
Beach-Camping	0	–	-0.0221	0.03
Beach-Cycling	0	–	-0.0573	0.02
Beach-Golf	0	–	-0.0736	0.01
Beach-Ski cross	0	–	-0.0233	0.01
Beach-Ski down	0	–	-0.1388	0.04
Camping-Cycling	0	–	-0.0163	0.01
Camping-Golf	0	–	-0.0203	0.01
Camping-Ski cross	0	–	-0.0354	0.01
Camping-Ski down	0	–	0.0494	0.04
Cycling-Golf	0	–	-0.0427	0.01
Cycling-Ski cross	0	–	-0.0039	0.00
Cycling-Ski down	0	–	0.0054	0.02
Golf-Ski cross	0	–	-0.0018	0.00
Golf-Ski down	0	–	0.0121	0.03
Ski cross-Ski down	0	–	-0.003	0.02

Note: The LR statistic ($q = 15$), for the models used ASU and NASU, amounts to 61.86 with a p-value of 2.53e-07.

4.3.4 Welfare measures

For both the ASU and NASU approach, we calculate the compensating variation following the approach presented by Vásquez and Hanemann (2008), calculations are made for two scenarios:

1. Individual variation: the price of only one alternative will be increased by one monetary unit, while the price of the other alternatives (6 remainings) will be increased by one monetary unit. For example, the price of Beach is increased by one unit, and the prices of Camping, Cycling, Golf Ski cross, and Ski down remain constant.

2. Variation in pairs:the price of two alternatives will be increased simultaneously in one monetary unit and at the same time the price of the other alternatives (5 remaining) will remain constant. For example, one unit increases the prices of Beach and Camping, and the prices of Cycling, Golf Ski cross, and Ski down remain constant.

Therefore, the hypothesis for evaluating differences between the calculation of both approaches corresponds to

$$\begin{aligned} H_0 &: CV_{j,ASU} = CV_{j,NASU}, \\ H_1 &: CV_{j,ASU} \neq CV_{j,NASU}, \end{aligned} \tag{4.7}$$

where CV_j corresponds to the CV of a change in the price of alternative (or alternatives) j through the ASU or NASU approach. Table 4.6 shows the results of the compensatory variation (mean) calculations for the different price changes. It is observed that the calculations of welfare measures through the NASU approach are lower than those obtained through the ASU approach, even in this case, the differences between both approaches can be significant (observing the t-values of the difference test.). For example, for the case where only the price of Ski Cross varies, when calculating the confidence interval of the CV through ASU approach we have that this is (1.59;1.75), then the confidence interval for the CV through the NASU approach is (1.41;1.55), from this it is observed that the confidence intervals are not contained within each other. On the other hand, the results for the maximum values of compensatory variations obtained, where it is observed that in all the scenarios of price variations there are significant differences between both approaches, this is the values obtained through the NASU approach are statistically lower than those of the ASU approach.

Table 4.6: Compensating Variation

ΔAlt.	Mean					Maximum				
	ASU		NASU		Hypothesis	ASU		NASU		Hypothesis
	est.	s.e.	est.	s.e.	t-statistic	est.	s.e.	est.	s.e.	t-statistic
Beach (B)	4.03	0.23	3.61	0.19	1.00	25.36	2.5	16.49	1.12	2.45
Camping (CA)	1.98	0.08	1.77	0.06	1.50	9.3	0.85	6.07	0.36	2.67
Cycling (CY)	2.44	0.11	2.08	0.09	1.80	12.96	1.23	7.91	0.5	2.92
Golf (G)	3.64	0.2	3.14	0.15	1.43	22.32	2.19	13.86	0.93	2.71
Ski cross (SC)	1.67	0.05	1.48	0.04	2.11	6.82	0.6	4.37	0.24	2.92
Ski down (SD)	3.23	0.17	2.68	0.12	1.9	19.11	1.85	11.26	0.74	3.03
B-CA	6.02	0.31	5.38	0.25	1.14	34.66	3.36	22.56	1.49	2.49
B-CY	6.48	0.35	5.7	0.28	1.24	38.33	3.73	24.41	1.63	2.6
B-G	7.68	0.44	6.76	0.34	1.18	47.68	4.71	30.35	2.05	2.56
B-SC	5.71	0.29	5.09	0.23	1.19	32.18	3.1	20.86	1.37	2.53
B-SD	7.27	0.4	6.29	0.31	1.38	44.47	4.36	27.75	1.86	2.69
CA-CY	4.43	0.2	3.86	0.15	1.63	22.27	2.08	13.99	0.86	2.82
CA-G	5.63	0.28	4.91	0.22	1.44	31.63	3.04	19.93	1.3	2.7
CA-SC	3.66	0.14	3.25	0.11	1.64	16.13	1.45	10.44	0.61	2.76
CA-SD	5.21	0.25	4.45	0.19	1.73	28.41	2.72	17.33	1.11	2.89
CY-G	6.09	0.32	5.23	0.25	1.51	35.29	3.42	21.78	1.43	2.79
CY-SC	4.12	0.17	3.57	0.13	1.83	19.79	1.82	12.28	0.74	2.93
CY-SD	5.68	0.29	4.77	0.22	1.78	32.08	3.1	19.17	1.24	2.97
G-SC	5.32	0.26	4.63	0.2	1.50	29.14	2.79	18.23	1.18	2.75
G-SD	6.88	0.38	5.83	0.28	1.59	41.43	4.06	25.12	1.67	2.85
SC-SD	4.91	0.23	4.16	0.17	1.88	25.93	2.46	15.63	0.99	2.99
All	17.02	0.87	14.78	0.68	1.45	95.89	9.25	59.98	3.91	2.73

Note: column ΔAlt. indicates the alternatives that changed in +1 unit the price. Standard errors are calculated through bootstrap.

Chapter 5

Conclusions

The use of choice experiments became popular in the context of economic valuation with contributions such as those of McFadden (1977) and Train (1998; 2003). However, researchers have focused more on calculating welfare measures than on understanding the incidence of how attributes entering the utility function might affect the calculation of welfare measures[1].

5.1 Contributions and implications

This paper aimed to evaluate the effect of the incorporation (or not) of complementarity patterns (in the preference structure) in the estimation of the MDC decision and welfare measures calculation. Through comparison strategies, for the forms ASU and NASU functions, it was found that (1) the terms of interaction between alternatives of choice have a significant impact on the consumer's decision, (2) there are no significant differences between the satiety and BMU parameters (3) there are significant differences (on more than one) between the calculation of welfare measures between the ASU and NASU approach.

5.1.1 Inference of flexible relations between alternatives

As described in the previous chapters, the ASU approach does not allow capturing patterns of complementarity between alternatives. That is, through the ASU approach, it is not possible to determine whether recreational activities Cycling, Hiking, Garden, and Photo are substitutes or complementary, because the modeling of the preference structure only allows them to be substituted for each other. However, through the extension to the NASU approach, significant relationships were found between the alternatives that are not the product of an initial restriction but are inferred from the flexibility of the

[1]Authors such as Torres et al. (2011) have delved into the incidence of preference structure in choice models, however, they do not extend to the case of MDC modeling with NASU approaches

preference structure modeling.

This understanding can be an orientation for policymakers. Following the case of recreational activities, a policy designer may be interested in designing policies to encourage recreational activities, for this he must choose how and which recreational activities to encourage. Therefore, if the ASU approach to policy design is followed, only activities will be considered (restrictively) as substitutes for each other, that is, if it encourages two recreational activities such as Camping and Beach, will not be able to identify previously whether individuals would consider attending only one activity (substitutes) or participating in both (complementary). Therefore, it is through the use of a NASU approach that the policymaker can guide the promotion of recreational activities that complement each other and then promote, separately, recreational activities that are substitutes. For example, in this paper it was found that the Campping and Beach activities have substitution relationships with each other, therefore it would be a mistake on the part of the policy designer to try to increase participation in Hiking through a promotion plan in the Beach activity because these two are not complementary, but the policy designer will not have this information through the use of an ASU approach.

5.1.2 Welfare measures

Due to the difficulties that arise in the estimation process when working with NASU functions, researchers have not been able to strongly explore the richness of these preference structures in the analysis of welfare measures, The analysis is therefore limited to the ASU approach, where alternatives will only be substitutes for each other.

However, in this work it was possible to work with at least one NASU functional form, which allowed comparing welfare measures obtained through the ASU and NASU approaches. Therefore, this work contributes to the understanding of the calculation of welfare measures with consideration of complementarity and substitution patterns.

Welfare measures were first estimated by conditioning the preference structure to substitution-only alternatives (ASU), then allowing flexibility to incorporate complementarity patterns (NASU) into the preference structure. The main results indicated that welfare measures that were calculated through the NASU approach may be, with some probability, significantly lower than those obtained by the ASU approach.

This result becomes relevant in the context of cost-benefit analysis because it is conditioned to the calculation of welfare measures. Therefore, if the investigator does not discriminate between the ASU and NASU approaches he could be overestimating welfare measures, which distorts the orientation of the cost-benefit analysis. In this sense, it will be the task of each of the researchers to evaluate the incidence of complementarity patterns in the calculation of welfare measures: in this paper, it was proposed to follow the Vásquez and Hanemann (2008) procedure, both for ASU and NASU approach, At the moment, it is the only way to estimate welfare measures through the NASU approach.

5.2 Limitations and directions

5.2.1 Complementary structure

This paper explored the estimation of the MDC decision through a NASU approach using the complementarity structure proposed by Bhat et al. (2015). However, there are other proposals to model the "interaction" structure between alternatives, such as those proposed by Musalem et al. (2013) and Pellegrini et al. (2019), which lead to additional work for researchers at the time of construction and estimation of the Jacobian matrix. However, these models have a transverse constraint which is the symmetry of complements.

The assumption of complement symmetry may be (or not) valid (or invalid) for some choice scenarios, therefore it may be recognized as a constraint in modeling the complementarity structure. Two alternatives have a relationship of asymmetric supplements when one depends more on the other, but individuals get more useful by consuming both, for example, in the context of recreational activities Hiking and Landscape Photography may present an asymmetrical complementarity relationship since we could assume that Landscape Photography depends more on the Walk because you need to explore different places to get good photographs.

Lee et al. (2010) explored the MDC decision through an asymmetric supplement approach, they consider cereals and milk as consumption alternatives, this modeling is limited to a choice set of two alternatives and does not allow the integration of substitution effects, However, this can be considered as a starting point, since the relationship between the problem raised in this paper and the integration of asymmetric complements has not been fully explored. Therefore, one of the directions to follow in modeling the MDC choice through a NASU approach is the integration of symmetrical, asymmetric, and substitution patterns.

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