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Extensiones del problema de Büchi a distintas estructuras y potencias más altas

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Capítulo 2

A survey on Büchi's problem : new presentations and open problems

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Abstract: In any commutative ring A with unit, *Büchi sequences* are those sequences whose second difference of squares is the constant sequence (2). Sequences of elements x_n satisfying $x_n^2 = (x+n)^2$ for some fixed x are Büchi sequences that we call *trivial*. Since we want to study sequences whose elements do not belong to certain subrings (e.g. for fields of rational functions $F(z)$ over a field F we are interested in sequences that are not over F) the concept of *trivial sequences* may vary. Büchi's Problem for a ring A asks whether there exists a positive integer M such that any Büchi sequence of length M or more is trivial.

We survey the current status of knowledge for Büchi's problem and its analogues for higher-order differences and higher powers. We propose several new and old open problems. We present a few new results and various sketches of proofs of old results (in particular: Vojta's conditional proof for the case of integers and a quite detailed proof for the case of polynomial rings in characteristic zero), and present a new and short proof of the positive answer to Büchi's problem over finite fields with p elements (originally proved by Hensley). We discuss applications to Logic (which were the initial aim for solving these problems).