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# ***A simulation-optimization approach for the fire stations location and vehicle assignment problem: A case study in the Concepcion Province, Chile.***

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# RESUMEN

***A simulation-optimization approach for the fire stations location and vehicle assignment problem: A case study in the Concepcion Province, Chile.***

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PROFESOR GUIA: Ph. D. Rodrigo Andrés De la Fuente Gallegos
PROGRAMA: Magíster en Ingeniería Industrial

Bomberos son una parte importante los servicios de emergencia al ser responsables de atender varias emergencias urbanas. Para atender estas de mejor forma, ellos deben planificar una adecuada localización de sus compañías y asignación de vehículos. Para apoyar esta toma de decisiones, proponemos un método iterativo de simulación-optimización que basado en parámetros de utilización previamente calculados actualiza la localización óptima de vehículos y compañías usando un modelo de programación lineal. Llamamos a este modelo el Facility Location and Equipment Emplacement Technique with Expected Coverage (FLEET-EXC), que considera multiples tipos de emergencias y vehículos, y una política de despacho que depende del tipo de región. Luego, modelo de simulación es ejecutado con las localizaciones y asignaciones obtenidas para actualizar los parámetros de utilización. Adicionalmente, el modelo de simulación usa un método de muestreo espacio-temporal que acopla un Kernel Density Estimator para el componente espacial y un proceso de arribo non-Stationary non-Renewal basado en un modelo de Markov-Mixture of Erlangs of Common Order para generar los tiempos entre-arribos para el componente temporal. Además, un conjunto de incertidumbre para los parámetros de utilización es obtenido de la simulación; por lo tanto, proponemos un modelo de optimización robusta para extender la formulación previa. Los principales resultados muestran que el método de muestreo propuesto logra una mejor representación del proceso de arribo de emergencias que aquellos generalmente usados en la literatura. Por otra parte, el procedimiento de simulación-optimización que usa el modelo de FLEET-EXC tiene un mejor desempeño que el modelo discrete FLEET, resultando en hasta 2% de mayor cobertura. Además, el modelo robusto también tuvo un mejor desempeño que el modelo discreto FLEET, pero tiene un desempeño variable al compararse con el FLEET-EXC. Sin embargo, el modelo robusto logra el menor tiempo de respuesta promedio cuando solo se consideran las emergencias bao el percentil 60.

**Palabras Claves**: Sistemas de Emergencia, Localización de Instalaciones, Simulación de Eventos Discretos, Optimización Robusta, Muestreo Espacio-temporal.

## ABSTRACT

***A simulation-optimization approach for the fire stations location and vehicle assignment problem: A case study in the Concepcion Province, Chile.***


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Firefighters are an important part of emergency service systems. Planners have to decide for the proper location of fire stations and the assignment of vehicles. To aid this decision making, we propose an iterative simulation-optimization approach that based on some precomputed utilization parameters updates the optimal location of vehicles and fire stations. First, we find fire station locations and vehicle assignments using the Facility Location and Equipment Emplacement Technique with Expected Coverage (FLEET-EXC) model, which considers multiple emergency and vehicle types, and a region-dependent dispatch policy. Second, we use a simulation model to find the utilization parameters from the previously computed solution. Then, if the obtained parameters deviate less than a desired error, the solution is maintained; whereas, on the contrary, these new parameters serve as input for the previous optimization model and a new solution is computed. Additionally, the simulation model uses a spatio-temporal sampling method that loosely couples a Kernel Density Estimator for the spatial component and a non-Stationary non-Renewal arrival process based on a Markov-Mixture of Erlangs of Common Order model to generate interarrival times for the temporal component. Moreover, an uncertainty set for the utilization parameters is obtained from the simulation; thus, we propose a robust optimization model to extend the previous formulation. The main results show that the proposed spatio-temporal sampling method achieves a better representation of the emergency arrival process than those generally used in literature. Moreover, the proposed models are compared to a discrete FLEET model that does not accounts for vehicles availability. The simulation-optimization procedure that uses the FLEET-EXC model performs better than the discrete FLEET model, resulting in up to 2% more coverage. On the other hand, the robust model also outperformed the discrete FLEET but has varying performance compared with the FLEET-EXC. Nonetheless, the robust model achieves the lowest average response time when only emergencies with response time below the 60th percentile are considered.



**Keywords**: Emergency Service Systems, Fire station location, Discrete event simulation, Robust optimization, Spatio-temporal sampling.

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# Chapter 1

# Introduction

Emergency services systems (ESS) perform one of the most complicated jobs today: they are responsible for saving peoples' lives. Every day, planners must dynamically allocate resources, which affects their capacity to respond to emergencies. Hence, information about where and when events occur is vital for resource location and assignment to improve the response time. Even though in most cases, information at an operational level is at hand for decision-makers, a broader scope for strategic planning is difficult to visualize if the tools are not available. Consequently, planners might not observe the benefits and costs of their long term decisions, and any sub-optimal location and vehicle assignment decision will lead to a waste of resources that have an alternative use.

Chile's firefighters (*Bomberos de Chile*) is a non-profit organization mainly integrated by volunteers. It is responsible for attending most of the civil emergencies such as residential fires, people's rescues, forest fires near urban areas, and accidents related to hazardous materials. Moreover, they are organized in fire departments, which are groups of fire stations assigned to a certain district and are administratively independent of each other. Also, each fire station has its group of volunteers and assigned vehicles, and they mainly fund its operations through donations and public funds. Due to this situation, it is difficult to articulate all the stakeholders involved in the strategic decision of where fire stations should be located and which vehicles assigned to them. Moreover, the lack of quantitative tools to forecast the expected performance of these decisions is an impediment to evaluate possible alternatives. Thereby, it is necessary to develop a standard methodology to support this decision-making process.

## 1.1 Problem description

The main objective when strategically planning fire station locations and resource assignment is to adequately cover as many potential areas where emergencies might occur as possible. This goal is difficult to seek because different decisions are involved such as which vehicle type to buy, which candidate location to use, or which current facility to relocate. Moreover, the coverage of a certain emergency depends mainly on the vehicle types it requires, adding more complexity to the decision-making process.

Firefighter operations can be described as follows. First, firefighters attend different types of emergencies, each one of these requiring a specific set of vehicles, i.e. a mix of basic and specialty vehicles. Basic ones are required to support every emergency that occurs, while specialty ones are assigned to the emergency they were designed for. Furthermore, the number of vehicles for each category that must be dispatched depends on the region where the emergency occurred, e.g. urban emergencies require more vehicles than rural emergencies, due to the higher risk of greater damage. Based on these requirements, a dispatch policy is designed to determine the set of vehicles that must be dispatch to a certain emergency. Moreover, a response time goal is determined for this set of vehicles to arrive at the scene to consider whether the emergency is properly served or not. This emergency coverage serves as a guideline for facility location and vehicle assignment decisions, giving the decision-maker a parameter to maximize. Although, we must keep in mind that firefighters' operations have a high degree of stochasticity associated with random variables such as the time spent serving emergencies, travel times, and the events' arrival rate, all of which might affect the availability of vehicles. Therefore, we must consider the utilization of vehicles to effectively compute the expected coverage of emergencies, and as a result, choose the optimal facility locations and vehicles assignment.

In this work, we address the facility location and vehicle assignment problem for firefighters as described above, where multiple emergency types must be attended using different vehicle types. Moreover, each emergency type requires a specific combination of vehicles depending on the region the event occurs, which we define as a region-dependent vehicle dispatch policy. This study aims to solve the problem by taking into account the stochastic behavior of firefighters' operations for strategic decision making. To accomplish this objective, we propose an iterative procedure that couples a mixed-integer linear programming (*MIP*) model with a discrete event simulation model to decide where to locate emergency facilities and which vehicle to assign, considering a more realistic evaluation of expected vehicles utilizations and their effect on emergency coverage under a previously specified response time. To characterize the stochasticity of these parameters, the simulation model includes a spatio-temporal arrival process for the generation of emergency events. Additionally, because utilization parameters are stochastic, they can be efficiently computed from the simulation outputs in the form of an uncertainty set, allowing us to evaluate a robust optimization approach as solving method by incorporating the set into the MIP model to account for uncertainty.

## 1.2 Contributions

The main contributions of this work are:

- Present an iterative simulation-optimization approach to solve the facility location and vehicle assignment problem with multiple demand types, different vehicle types, and region-dependent dispatch policies.
- Present a novel loosely coupled spatio-temporal sampling method for the generation of emergency events.
- Develop a computationally efficient method to compute travel times from actual point-to-point that uses the actual street network with time-varying speeds.

The rest of this work is organized as follows. First, a review of the state-of-the-art in facility location problems using simulation is presented in Chapter 2. Next, in Chapter 3, the MIP model is described as well as the proposed simulation model. This section also explains the spatio-temporal sampling methodology for the events' arrival process implemented in the simulation and the iterative procedure that couples the MIP model with the simulation. Then, in Chapter 4, a case study was conducted on the Concepcion province, Chile, to solve the fire stations' location and vehicle assignment problem on twelve fire departments. Moreover, Chapter 5 presents the main results and discussions of the comparison between different location policies' optimal results and their effect on the coverage of emergencies. Finally, in Chapter 6, the main conclusions and extensions of this work are presented.

# Chapter 2

# Literature review

In this section, we present different studies related to the facility location and vehicle assignment problem where simulation was included in the solving approach. To begin with, in Table 2.1 we compare these works to identify differences concerning the solution approach. We distinguish two primary uses for a simulation model. First, simulation can be considered a part of an optimization procedure, which is commonly known as *Simulation Optimization*. Here, a simulation model may be used to compute the fitness function in a Metaheuristic (Genetic algorithms, Search algorithm) or, as an evaluation module coupled with linear programming (LP) models in a heuristic procedure. Second, a simulation can be used as a descriptive method to benchmark the location and assignment solutions that may be obtained from different LP models. On the other hand, we classify the type of simulation model used in each work as a discrete event simulation (DES) or as agent-based simulation (ABS). Next, to evaluate the real application of each study, we also considered

**Table 2.1:** Use of simulation on ESS location models in the presented literature

| Reference | Use of Simulation | | | Simulation type | | Includes | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Optimization Procedure | | Benchmark | DES | ABS | Case Study | | | GIS |
| | Metaheuristic | with LP | | | | Ambulances | Fire stations | Helicopters | |
| Savas (1969) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| Hendrick et al. (1975) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ☐ | ⊠ | ☐ | ☐ |
| Goldberg et al. (1990) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| Yang et al. (2004) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ☐ | ⊠ | ☐ | ⊠ |
| Aringhieri et al. (2007) | ☐ | ☐ | ⊠ | ☐ | ⊠ | ⊠ | ☐ | ☐ | ☐ |
| Haghani and Yang (2007) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ⊠ |
| Bjarnason et al. (2009) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ☐ | ⊠ | ☐ | ☐ |
| Silva and Pinto (2010) | ⊠ | ☐ | ☐ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| Lee et al. (2012) | ☐ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ | ⊠ | ⊠ |
| Aboueljinane et al. (2012) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| McCormack and Coates (2015) | ⊠ | ☐ | ☐ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| Jagtenberg et al. (2015) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| Ünlüyurt and Tunçer (2016) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| Karatas et al. (2017) | ☐ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ | ⊠ | ☐ |
| Enayati et al. (2018) | ☐ | ☐ | ⊠ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ☐ |
| **Our work** | ☐ | ⊠ | ☐ | ⊠ | ☐ | ☐ | ⊠ | ☐ | ⊠ |

☐: not included ⊠: included.

**Table 2.2:** ESS location models in the presented literature

| Reference | Spatio-temporal sampling | | | | | | Travel times | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Available data | Spatial | | Temporal | | | Street network | | | Speed | | |
| | | Uniformly | KDE | Poisson | NHP | NPNH | Actual road | Rectilinear | Euclidean | Regression | Constant | Varying |
| Savas (1969) | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ |
| Hendrick et al. (1975) | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ |
| Goldberg et al. (1990) | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ |
| Yang et al. (2004) | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ | ☒ | ☐ | ☐ | ☐ | ☒ | ☐ |
| Aringhieri et al. (2007) | ☒ | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☒ | ☐ |
| Haghani and Yang (2007) | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ | ☒ |
| Bjarnason et al. (2009) | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☐ | ☐ | ☒ |
| Silva and Pinto (2010) | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☐ | ☒ |
| Lee et al. (2012) | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒* | ☐ | ☒ | ☐ |
| Aboueljinane et al. (2012) | ☐ | ☒ | ☐ | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ | ☒ |
| McCormack and Coates (2015) | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ | ☒ |
| Jagtenberg et al. (2015) | ☐ | ☐ | ☐ | ☒ | ☐ | ☐ | ☒ | ☐ | ☐ | ☐ | ☒ | ☐ |
| Ünlüyurt and Tunçer (2016) | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒ | ☐ |
| Karatas et al. (2017) | ☐ | ☒ | ☐ | ☒ | ☐ | ☐ | ☐ | ☐ | ☒* | ☐ | ☐ | ☒ |
| Enayati et al. (2018) | ☒ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☒* | ☒ | ☐ | ☐ |
| **Our work** | ☐ | ☐ | ☒ | ☐ | ☐ | ☒ | ☒ | ☐ | ☐ | ☐ | ☐ | ☒ |

☐: not included ☒: included.

if the work included a case study, the type of service for which it was developed, and if they employed Geographic Information Systems (GIS) for the data analysis.

Table 2.2 identifies two particular aspects of the simulation that were considered important when modeling ESS. On the one hand, it classifies the methodology used for the representation of the spatio-temporal behavior of the events' arrival process. First, the spatial distribution of events may be represented as a uniform distribution (mainly by using square grids), or as a smoother distribution, such as a Kernel Density Estimator (KDE). Then, for the modeling of the temporal component, the following arrival processes types were considered: Poisson, non-Homogeneous Poisson (NHP), and non-Homogeneous non-Poisson (NHNP). On the other hand, Table 2.2 also compares how the authors computed travel times from facilities to emergencies by modeling distance and speed variables.

## 2.1 Facility location problems for emergency systems

Numerous studies in the past decades have addressed the facility location problem for ESS. Although current facility location models present several formulations, most of them are based on the following three models. First, the *P-Median Location Problem - PMLP* presented by Hakimi (1964), whose objective is to minimize the demand weighted by its distance to one of $p$ opened facilities. Second, the *Set Covering Location Problem - SCLP* by Toregas et al. (1971) that minimizes the number of facilities to locate, securing that all demand nodes are being served under a coverage threshold. Third, the *Maximal Covering*

*Location Problem - MCLP* by Church and ReVelle (1974) that maximizes the coverage of demand under a covering threshold by locating a fixed number of facilities. Applications for siting fire stations were developed using an SCLP by Plane and Hendrick (1977) in Denver, Colorado and by Schreuder (1981) in Rotterdam, Netherlands. Moreover, a study was developed for ambulance location using an MCLP by Eaton et al. (1985) in Austin, Texas, and by Van den berg et al. (2017) to locate fire stations in Amsterdam, Netherlands.

Even though the previous models addressed primary location problems, their deterministic nature and oversimplification left space for improvement. Extensions for the MCLP were developed by Schilling et al. (1979) to consider vehicle assignment (*Facility Location & Equipment Emplacement Technique - FLEET*), and by Daskin (1983) to consider facility availability and expected coverage (*Maximal Expected Covering Location Problem – MEXCLP*). Furthermore, probabilistic components were included in location models by securing a service level for each demand node in ReVelle and Hogan (1989) with the *Maximal Availability Location Problem – MALP* and its extension for the FLEET in ReVelle and Marianov (1991).

Another branch of extensions was developed to include the stochastic behavior of facilities and vehicles by using the *Hypercube Queueing Model - HQM* presented in Larson (1974). The HQM is a descriptive model based on Queueing Theory where two states are defined for each server: available and busy. Having a system with $N$ servers, a system's state can be described as a vertex of a $N$-dimension hypercube that represents a particular combination of servers' states. The transition rates between states are obtained from the servers' spatial distribution and dispatch rules. From this model, servers' availability is computed by an approximation procedure presented in Larson (1975). Applications of the HQM on ambulances location can be found on Brandeau and Larson (1986) in Boston, Massachusetts, and Mobin et al. (2015), in Teheran, Iran. Because the HQM is a descriptive model, complimentary use LP models and metaheuristics were developed to solve the facility location problem for emergency services. Studies with LP models can be found on Batta et al. (1989) with the adjusted MEXCLP *(AMEXCLP)*, McLay (2009) with the MEXCLP2 for two types of servers, and Chevalier et al. (2012) with an MCLP to first locate fire stations in Belgium, and later assign crews with an HQM. Applications using genetic algorithm were presented in Saydam and Aytuğ (2003), Iannoni et al. (2009), and Toro-Díaz et al. (2013), while Rajagopalan et al. (2008) presents a tabu search procedure.

## 2.2 Simulation models for facility location

Another tool used to describe stochastic components of emergency service operations is a simulation. Currently, three simulation paradigms exist: DES, ABS, and system dynamics (SD). As with the HQM, simulation can be employed as a description method to obtain response metrics from a specific set of facilities. Based on this application, we can distinguish two main methodologies to solve the facility location problems: i) the use of simulation to compare previously obtained location solutions, and ii) the use of simulation as part of an optimization procedure (heuristic or metaheuristic).

The first approach implies that alternative locations are computed (by solving either an LP model or a heuristic method) and compared among them and the current layout to obtain insights based on predefined performance metrics. Applications of this methodology are first presented on Savas (1969), which was the first study to use a simulation to assess ambulance location in the city of New York. Later on, Hendrick et al. (1975) used a simulation model to evaluate different configurations of fire stations in Denver, Colorado, and Goldberg et al. (1990) compared ambulance locations in Tucson, Arizona, both using DES. Yang et al. (2004) proposed a multi-objective model for fire station redistricting, and simulated the resulting locations with DES to compare them with the current layout. Aringhieri et al. (2007) presented an ABS to contrast solutions from a MIP model for ambulance location in Milan, Italy. Afshartous et al. (2009) used DES to obtain a robust solution for the US Coast Guards air station location by comparing two MIP models. Bjarnason et al. (2009) proposed an optimization algorithm for fire stations dispatch policies and evaluated them using DES. More applications include simulating emergency operations to experiment with different operational decisions as in Aboueljinane et al. (2012), for an ambulance system in Val-de-Marne, France, and for firefighters in Kuwait as presented in Aleisa and Savsar (2013). Recently, Jagtenberg et al. (2015)

presented a heuristic to locate ambulances in polynomial time and compared the results with an LP model using DES. Ünlüyurt and Tunçer (2016) did a similar study where a comparison between two LP models was made using a DES for ambulances in Istanbul, Turkey. Finally, Enayati et al. (2018) proposed a real-time approach to maximize coverage at the minimum possible travel times by redeploying ambulances in Mecklenburg County, North Carolina. They solved this problem by combining two computational inexpensive models, and compared the obtained results with the current location policy, using a DES model.

The second approach for solving location problems with simulation consists of using the outputs of a simulation model as inputs of an optimization procedure such as metaheuristics (genetic algorithms, simulated annealing), heuristic algorithm, or iterative procedures. Silva and Pinto (2010) modeled a DES for ambulance operations in Belo Horizonte, Brazil, and used OptQuest, a popular optimization module implemented on most simulation software that uses metaheuristics searches, for optimal location. Lee et al. (2012) proposed an LP model for air ambulance locations that considered the availability of helicopters. This parameter was calculated from a DES, and an iterative procedure is used to update the objective function based on the previously obtained solution. Later, McCormack and Coates (2015) coupled simulation with a genetic algorithm for ambulance location in London, England. Finally, Karatas et al. (2017) presented a three-module optimization procedure to allocate search and rescue helicopters. The first module solved an LP model. Then, the second module simulated the location results. After this, the third module generated alternative plans using the simulation outputs. An iterative procedure is implemented, where the best alternatives are selected and simulated to generate alternative plans until no significant changes are made.

Based on the previous studies, the main challenge when using simulation for solving facility location problems is to obtain representative output parameters from the simulation model. Although the more complex a simulation model is the better results it produces, a trade-off between complexity and solving time must be made to avoid non-viable models. Thus, the primary goal is to seek realism while maintaining computational efficiency. In this work, we develop a customer-server model that simulates the emergency arrival process with the highest level of granularity, considering for each emergency event its specific geographic location and occurrence time. Moreover, we used the actual street network with time-varying speeds for each street type to compute travel times from facilities to demand points. The resulting simulation model is computationally inexpensive, allowing to develop a simulation-optimization approach to solve the facility location and vehicle assignment problem presented above.

## 2.3 Spatio-temporal simulation

A problem that every customer-server simulation model has to address is the representation of the arrival process. On emergency location problems, this process is described by a temporal and spatial distribution. Although most studies can fit collected data to a known probabilistic distribution (such as a Poisson distribution) for the first component, describing spatial behavior presents a challenge due to the complexity of geographic representation and data precision. Aggregation of events into larger and simpler subregions (usually square grids) is a common solution to this problem (Aringhieri et al. (2007), McCormack and Coates (2015), Karatas et al. (2017)). In recent years, the use of geographic information systems (GIS) to analyze geo-data presents an opportunity to add realism for spatio-temporal simulation. Peleg and Pliskin (2004) used GIS to pinpoint calls and create response time polygons to evaluate coverage from ambulances location. Later, in Haghani and Yang (2007) GIS locations of the historical spatial distribution of emergency calls were fitted using Arena Analyzer. A DES was used to evaluate various dispatching policies obtained from an LP model. Finally, Asgary et al. (2010) developed spatial, temporal and spatio-temporal analyses to determine the causality of residential fire incidents, while using a KDE to model the spatial component of emergencies.

As for the temporal component, the first complexity encountered when trying to simulate a process is that the arrival rate $r$ seems to be time-varying, which has some logic if we are representing the behavior of an emergency service where calls frequency change during the day. If the coefficient of variation ($cv$) is equal to one, this situation can be modeled as a non-homogeneous Poisson process (NHPP) using a time-varying rate $r(t)$, and sampling from it using the inverse of the integrated-rate function as shown in Law and Kelton (2000). However, in real-life systems $cv$ is rarely equal to one, and the NHPP turns to be inadequate. Thus, the arrival process can be characterized as a non-homogeneous non-Poisson (NHNP), where similarly to the NHPP, the process has a time-varying rate, but the $cv$ differs substantially from one. Gerhardt and L. Nelson (2009) provided methods for fitting and simulating an NHNP. Later on, Nelson and Gerhardt (2011) generalized this method to facilitate the generation of non-stationary non-renewal (NSNR) arrivals. They propose the Markov-Mixture of Earlangs of Common Order sampling method for generating NSNR arrival processes, by defining an empirical time-varying arrival rate, a parameter of correlation between arrivals, and the squared coefficient of variation of the arrival process.

Despite the contributions of the previous studies, limitations are present. First, the use of square grids to aggregate events points distorts the actual spatial distribution of emergencies, affecting the estimation of the response time which is a key parameter in the aforementioned models. Second, the representation of the road network fails to consider time-varying street speeds and actual distances, also affecting the resulting response time. Third, the absence of correlation on the modeling of the emergency arrival process may produce a misleading estimation of the actual utilization of vehicles. In this work, we addressed these limitations to properly obtain the average utilization of vehicles to compute the expected coverage of emergencies to solve the facility location and vehicle assignment problem for firefighters.

As mentioned before, the primary goal of a simulation model is to seek realism while maintaining computational efficiency. To achieve this, we propose the following spatio-temporal sampling method. First, we use the Markov-MECO introduced by Nelson and Gerhardt (2011) for generating NSNR arrival processes due to its easiness to model any arrival process (including the correlation between interarrival times) by using empirical data. Second, the spatial component is modeled with KDE to avoid geographic misrepresentation due to spatial aggregation. Finally, both distributions are loosely coupled to model the spatio-temporal distribution of emergencies. As far as we know, this methodology is novel for spatio-temporal simulation of complex emergency arrival process. Based on the reviewed studies we conclude that, to the better of our knowledge, these work contributions are: i) proposing an iterative simulation optimization procedure including a MIP model for the fire stations location and vehicle assignment problem, ii) develop a computationally efficient method to compute travel times from actual point-to-point that uses the actual street network with time-varying speeds, and iii) the use of a novel loosely coupled spatio-temporal sampling method using KDE, NSNR arrival process, and GIS data to simulate the emergencies arrival process.

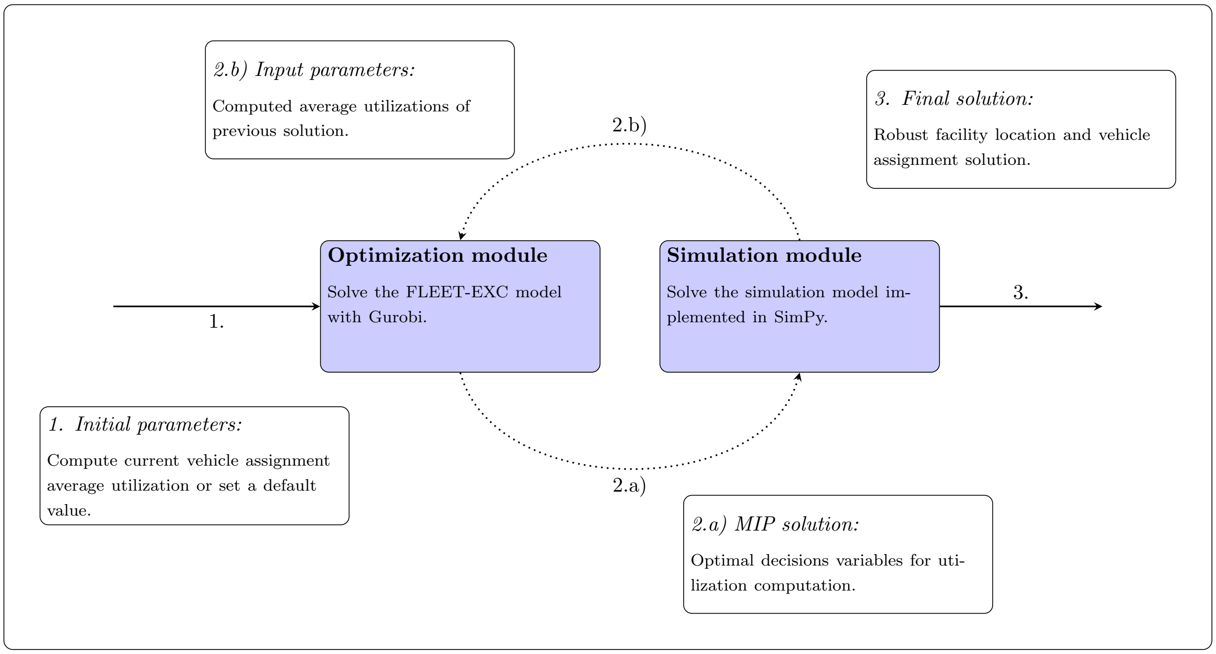
# Chapter 3

# Methodology

To solve the facility location and vehicle assignment problem addressed in this study, it is clear that the optimal solution depends heavily on the vehicles' utilization, which in turn affects the expected coverage of demands. Therefore, different sets of these parameters may produce non-identical location and assignment solutions, as a consequence, variability is induced into the problem when deciding which set of utilization parameters to use. Moreover, we must consider that an optimal solution for a set of utilization parameters (which may be obtained from current facilities locations and vehicle assignments), may produce a different set of utilization parameters as a result of the random nature of executing a simulation program. Thus, the aim of the following methodology is to obtain a set of vehicle utilization parameters from an optimal solution that deviates as little as possible from the initial parameters used to compute it. An iterative approach is proposed and shown in Figure 3.1 to update these parameters until they converge into a unique solution.

First, a MIP model is developed to solve the facility location and vehicle assignment problem for firefighters' operations. This model takes into account different vehicle types, multiple demand types and region-dependent vehicles dispatching rules. Then, a simulation model computes the average vehicles' utilization from the MIP optimal solution. If parameters deviate less than a predefined error ($\epsilon = 0.005$ is used in this study), then the resulting facility location and vehicle assignment decision are maintained. Conversely, if this condition is not met, the simulated parameters serve as input to update the MIP model and a new optimal solution is computed. Additionally, it is important to mention that even though this method results in a single solution in most cases, it does not ensure convergence. Moreover, it may be the case that the iterative procedure encounters a loop between two or more sets of parameters. Thus, we considered a predefined number of iter-

**Figure 3.1:** Iterative procedure.




ations as an additional stopping criterion to avoid excessive iteration. In practice, only few experiments did not meet the first stopping criteria. Nevertheless, the obtained solutions are optimal for a set of realistic utilization parameters and are suitable for decision making.

In the following subsection, we present the notation used in this work. Next, the MIP model used to solve the facility location and vehicle assignment problem by maximizing the expected coverage of emergencies is presented. Then, a description of the discrete event simulation model that calculates the average utilization of vehicles is given. Specifically, in this subsection, the modeling of the emergency arrival process, considering both its temporal and spatial components, is shown. Finally, an alternative robust optimization MIP that considers an uncertainty set for utilization parameters is presented.

## 3.1 Notation

**Sets:**

- $N$ set of fire departments.
- $H$ set of fire departments types.
- $I$ set of demand nodes.
- $I^h$ set of demand nodes located on a fire department of type $h \in H$, with $I^h \subset I$.
- $n_i$ fire department where demand node $i \in I$ is located.
- $J$ set of candidate and current fire station locations.
- $J^{'}$ subset of existing fire station locations.
- $J^{''}$ subset of candidate fire station locations.
- $J^{'}_n$ subset of locations with existent fire station on fire department $n \in N$, with $J^{'}_n \subset J$.
- $J^{''}_n$ subset of candidate fire station locations in fire department $n \in N$, with $J^{''}_n \subset J$.
- $L$ set of emergency types.
- $K$ set of vehicle types.
- $K^e$ subset of specialty vehicle types, with $K^e \subset K$.
- $K^l$ subset of vehicle types that an emergency of type $l \in L$ needs, with $K^l \subset K$.
- $b$ basic vehicle type.
- $e$ specialty vehicle type needed to attend emergency type $l \in L$ with $e \in K^e \cup K^l$
- $G$ set of amount of vehicles that may cover a demand.

**Parameters:**

- $q$ number of new vehicles per type to assign to a new fire stations.
- $d_{il}$ frequency of emergencies of type $l \in L$ on demand node $i \in I$.
- $v_{kj}$ number of existing vehicles of type $k \in K$ currently at $j \in J^{'}$.
- $\rho_{kn}$ average utilization of vehicles type of $k \in K$ on fire department $n \in N$.
- $c_{kn}$ amount of vehicles of type $k \in K$ available to assign at fire department $n \in N$.

- $t_{ji}$ travel time from candidate fire station location $j \in J$ to demand node $i \in I$.
- $t_{max}$ response time threshold to cover a demand node.
- $m_{klh}$ amount of type $k \in K$ vehicles that must be dispatched to serve an emergency of type $l \in L$ on fire department type $h \in H$ .
- $m_{gklh}$ amount $g \in G$ of type $k \in K$ vehicles that must be dispatched to serve an emergency of type $l \in L$ on fire department type $h \in H$.
- $a_{ji}$ coverage parameter.

  1, if $t_{ji} < t_{max}$, with $j \in J$, $i \in I$

  0, otherwise

- $p_n$ number of existing fire stations to maintain at fire department $n \in N$.
- $p_n^{reloc}$ number of fire stations to relocate at fire department $n \in N$.
- $p_n^{new}$ number of new fire stations to open at fire department $n \in N$.
- $M$ large number.
- $Q(c_{kn}, \rho_{kn}, g_k)$

  factor that quantifies the correction to the probability of obtaining $g_k$ busy vehicles of type $k$ followed by an available one when assuming that vehicles operate independently (Larson (1975)). Depends on the amount of vehicles $c_{kn}$, the average utilization $\rho_{kn}$ and the number of previously preferred vehicles $g_k$.

**Variables:**

- $z_j$ binary variable indicating whether a fire station is opened or not.

  1, if a fire station is opened on candidate location $j \in J$.

  0, otherwise

- $y_{il}$ binary variable indicating coverage on a demand node.

  1, if emergency type $l \in L$ on demand node $i \in I$ is covered by the necessary basic and specialty vehicles.

0, otherwise

- $y_{ilg_bg_e}$ binary variable indicating coverage on a demand node.

  1, if emergency type $l \in L$ on demand node $i \in I$ is covered by $g_b \in G$ basic vehicles and by $g_e \in G$ speciality vehicles with $e : e \in K^l \bigcap K^e$.

  0, otherwise

- $w_{ilk}$ binary variable indicating the amount of vehicles covering a demand node.

  1, if demand node $i \in I$ of emergency type $l \in L$ is covered by the necessary vehicles of type $k \in K$.

  0, otherwise

- $w_{ilgk}$ binary variable indicating the amount of vehicles covering a demand node.

  1, if demand node $i \in I$ of emergency type $l \in L$ is covered by at least $g \in G$ vehicles of type $k \in K$.

  0, otherwise

- $x_{kj}$ number of vehicles of type $k \in K$ assigned to candidate fire station location $j \in J$.

- $s_{j'j''}$ binary variable indicating relocation of a fire station.

  1, if a fire station is relocated from $j' \in J_n^{'}$ to $j'' \in J_n^{''}$.

  0, otherwise

- $t$ auxiliary variable for robust model.

## 3.2 Optimization model

In this work, the *Facility Location and Equipment Emplacement Technique with Expected Coverage (FLEET-EXC)* model is presented to solve the fire station's location and vehicle assignment problem. This model considers the location of new fire stations and/or relocation of current ones on each fire department, as well as the assignment of basic and specialty vehicles on each fire station. Moreover, the objective of the *FLEET-EXC* is to maximize the expected coverage of demand, considering the average utilization of vehicles. Furthermore, this model also incorporates the coverage of multiple emergency types, where each one requires a specific set of basic and specialty vehicles. To gradually increase the complexity of the model, we first introduce the following deterministic *FLEET* model for fire stations locations and vehicle assignment:

**Discrete FLEET:**

$$\text{Maximize} \sum_{i \in I} \sum_{l \in L} d_{il} y_{il g_b g_e} \tag{3.1}$$

subject to:

**Vehicles and coverage**

$$m_{klh} w_{ilk} \leq \sum_{j \in J} a_{ji} x_{jk}, \qquad \forall i \in I^h, \forall l \in L, \forall k \in K^l, \forall h \in H \tag{3.2}$$

$$y_{il} \leq w_{ilk}, \qquad \forall i \in I, \forall l \in L, \forall k \in K^l \tag{3.3}$$

**Fire station location**

$$\sum_{j \in J'_n} z_j = p_n - p_n^{reloc}, \qquad \forall n \in N \tag{3.4}$$

$$\sum_{j \in J''_n} z_j = p_n^{new} + p_n^{reloc}, \qquad \forall n \in N \tag{3.5}$$

$$\sum_{j' \in J'_n} \sum_{j'' \in J''_n} s_{j',j''} = p_n^{reloc}, \qquad \forall n \in N \tag{3.6}$$

**Relocation links**

$$s_{j'j''} \leq \frac{z_{j''} - z_{j'} + 1}{2}, \qquad \forall j' \in J'_n, \forall j'' \in J''_n, \forall n \in N \tag{3.7}$$

$$\sum_{j' \in J'_n} s_{j',j''} \leq z_{j''}, \qquad \forall j'' \in J''_n, \forall n \in N \tag{3.8}$$

$$\sum_{j'' \in J''_n} s_{j',j''} \leq 1 - z_{j'}, \qquad \forall j' \in J'_n, \forall n \in N \tag{3.9}$$

**Vehicle assignment**

$$x_{kj''} \leq \sum_{j' \in J'_n} v_{kj'} s_{j'j''} + q(z_{j''} - \sum_{j' \in J'_n} s_{j'j''}), \qquad \forall k \in K, \forall j'' \in J''_n, \forall n \in N \tag{3.10}$$

$$\sum_{k \in K^e} x_{kj''} \leq M \sum_{j' \in J'_n} s_{j'j''} + q, \qquad \forall j'' \in J''_n, \forall n \in N \tag{3.11}$$

$$x_{kj} = v_{kj} z_j, \qquad \forall j \in J', \forall k \in K \tag{3.12}$$

**Variables domain**

$$x_{kj} \in \mathbb{Z}_+, \qquad \forall k \in K, \forall j \in J \tag{3.13}$$

$$u_{jk} \in \{0,1\}, \qquad \forall j \in J, \forall k \in K \tag{3.14}$$

$$z_j \in \{0,1\}, \qquad \forall j \in J \tag{3.15}$$

$$y_{il} \in \{0,1\}, \qquad \forall i \in I, \forall l \in L \tag{3.16}$$

$$w_{ilk} \in \{0,1\}, \qquad \forall i \in I, \forall l \in L, \forall k \in K^l \tag{3.17}$$

The objective function (3.1) maximizes the emergency coverage considering the average utilization of vehicles on each fire department. First of all, constraints set (3.2) link the assigned vehicles under the time threshold with the auxiliary coverage variables. Then, constraints sets (3.3) link the coverage variable $y_{il}$ with the auxiliary coverage variable $w_{ilk}$ to account for combine coverage of basic and specialty vehicles. Next, constraints sets (3.4), (3.5) and (3.6) set the number of fire stations to relocate, the number of current facilities to keep, and the number of new fire stations for each fire department. Moreover, relocation variable $s_{j'j''}$ are linked with location variable $z_j$ to identify relocated facilities from new fire stations by adding constraints sets (3.7), (3.8) and (3.9). Additionally, constraint set (3.10) fixes the number of vehicles to assign at a candidate node. If the opened facility is a new fire station, then a maximum of $q$ vehicles for each type can be assigned. Furthermore, the number of specialty vehicles assigned to a new fire station is bounded in constraints set (3.11). On the contrary, if it is a relocated facility, the same number of vehicles that belonged to the original fire station must be assigned. Then, constraint set (3.12) maintain current vehicle assignment on fire stations that remain in the same place. Finally, constraints (3.13) - (3.17) specify domains of the decision variables.

Additionally, even though the previous model solves the facility location and vehicle assignment problem, it overestimates the demand coverage because it assumes that emergencies can be attended at any time (vehicles are always available). However, in reality, vehicles remain busy for a fraction of the day which affects their actual capacity to attend emergencies. Thereby, the *FLEET-EXC* extends the previous model to maximize the expected coverage of emergencies by taking into account the average utilization of vehicles.

**FLEET-EXC:**

$$\text{Maximize} \sum_{i\in I}\sum_{l\in L}\sum_{g_b\in G}\sum_{g_e\in G} d_{il} y_{ilg_bg_e} * Q(C_{bn_i}, \rho_{bn_i}, g_b-1)(1-\rho_{bn_i})\rho_{bn_i}^{g_b-1} * Q(C_{en_i}, \rho_{en_i}, g_e-1)(1-\rho_{en_i})\rho_{en_i}^{g_e-1} \quad (3.18)$$

subject to:

$(3.4) - (3.15)$

**Vehicles and coverage**

$$m_{gklh} w_{ilgk} \leq \sum_{j\in J} a_{ji} x_{jk}, \quad \forall i \in I^h, \forall l \in L, \forall k \in K^l, \forall h \in H, \forall g \in G \quad (3.19)$$

$$y_{ilg_bg_e} \leq w_{ilgb}, \quad \forall i \in I, \forall l \in L, \forall g_b, g_e, g \in G \quad (3.20)$$

$$y_{ilg_bg_e} \leq w_{ilge}, \quad \forall i \in I, \forall l \in L, \forall g_b, g_e, g \in G \quad (3.21)$$

**Variables domain**

$$y_{ilk_bk_e} \in \{0,1\}, \quad \forall i \in I, \forall l \in L, \forall k_b, k_e \in G \quad (3.22)$$

$$w_{ilkg} \in \{0,1\}, \quad \forall i \in I, \forall l \in L, \forall k \in K^l, \forall g \in G \quad (3.23)$$

The objective function (3.18) maximizes the expected coverage of demand by considering the joint availability of both basic and specialty vehicles, adjusted by Larson's independence correction factor (Larson (1975)). Constraint (3.20) and (3.21) link the auxiliary coverage variable with coverage variable $y_{ilg_bg_e}$, securing that demand coverage is accomplished by the required number of both basic and specialty vehicles. Finally, constraints (3.22) and (3.23) define the new decision variables domains.

## 3.3 Robust optimization

Moreover, even though the FLEET-EXC model represents a more realistic version of the problem at hand, it still depends on the utilization-of-vehicles parameters used to estimate the expected coverage of emergencies. Consequently, variations on these parameters might affect the optimal solution. Additionally, in reality, this stochasticity is inheriting to emergency services operation, which produces the dilemma of which parameters to use. Robust linear optimization, as proposed by Soyster (1973), is an approach that considers a level of uncertainty related to unknown parameters by defining a set of parameter realizations to be included on constraints of the mathematical model. Then, the model is optimized by considering all these realizations, making its results more robust against unexpected variations of these parameters. In our case, the set of parameters is defined as uncertainty set $U = \{\theta : \{\theta_{kn}, \forall k \in K \; \forall n \in N\}\}$, which corresponds to the utilization parameters per vehicle type for each district of the FLEET-EXC model. However, based on Formulation 2, these parameters are located in the objective function, but the robust approach requires the uncertainty set to be incorporated on the constraints. Therefore, we must reformulate the FLEET-EXC to the following model:

**Robust FLEET-EXC:**

$$\text{Maximize} \quad t \tag{3.24}$$

subject to:

$$\sum_{i\in I}\sum_{l\in L}\sum_{g_b\in G}\sum_{g_e\in G} d_{il} y_{ilg_bg_e} * Q(C_{bn_i}, \theta_{bn_i}, g_b - 1)(1-\theta_{bn_i})\theta_{bn_i}^{g_b-1}$$
$$* Q(C_{en_i}, \theta_{en_i}, g_e - 1)(1-\theta_{en_i})\theta_{en_i}^{g_e-1} \geq t \qquad \forall \theta \in U \tag{3.25}$$

$$(3.4) - (3.15), (3.19) - (3.23)$$

In this study, the uncertainty set $U$ is computed by running a simulation model for the desired number of replicates per scenario. Despite that a large set of replicates is needed to account for the stochasticity of the parameter, we must consider that adding constraints to the MIP model increases its computational difficulty, and consequently its solving time. Therefore, the convex hull of $U$ is computed, and we define $U'$ as the set of all points of utilization parameters $\theta$ that lies on the convex hull, which reduces the number of constraints to add. Nonetheless, to generate the convex hull of a set of $n-$dimensional parameters, a minimum of $n+1$ observations are required. To avoid running too many replicates (which is computationally expensive), principal component analysis (PCA) is applied over a smaller set of sampled data to reduce its dimensionality. PCA is a linear transformation that is employed to project data into a lower-dimensional subspace and ensures that the new coordinates preserve most of the variation of the original data. Moreover, coordinates are arranged such that the greatest variation is represented by the first coordinate, and the rest is sorted in descending order. Thus, first, we use this method to reduce the dimensionality of our available data while maintaining its primary variability. Next, we compute the convex hull on the lower-dimension dataset, and, the vertices of this convex hull ($\theta' \in U'$) can be reprojected to the original space and the subset of $\theta$, that correspond to the $\theta'$ in the lower subspace, is used to build the new constraints for the robust model.

## 3.4 Discrete event simulation model

A discrete event simulation (DES) model was developed to estimate both the average utilization of vehicles for the *FLEET-EXC* model and the uncertainty set $U$ for the robust formulation. A fire station location and vehicle assignment solution is used as an input for the simulation model, among with firefighters operation variables such as emergency service expected rates and travel times between facilities and events. Next, the average utilization of vehicles is calculated by replicating emergency attendance by firefighters, following an established vehicle dispatch policy. Then, the resulting output of the simulation model serves as an input to the previously mentioned MIP model.

Although multiple simulation models describing firefighters' operations have been presented in recent years, the challenge of increasing realism without losing computational efficiency arises. Moreover, the granularity of geographic data and the availability of statistical tools, allow us to improve the representation of the emergency arrival and vehicle dispatch processes, enhancing the model's validity for stakeholders. Therefore, the following DES model aims to represent firefighters' operations as realistic as possible, considering available GIS tools for street networks analysis and spatiotemporal distribution adjustment techniques.

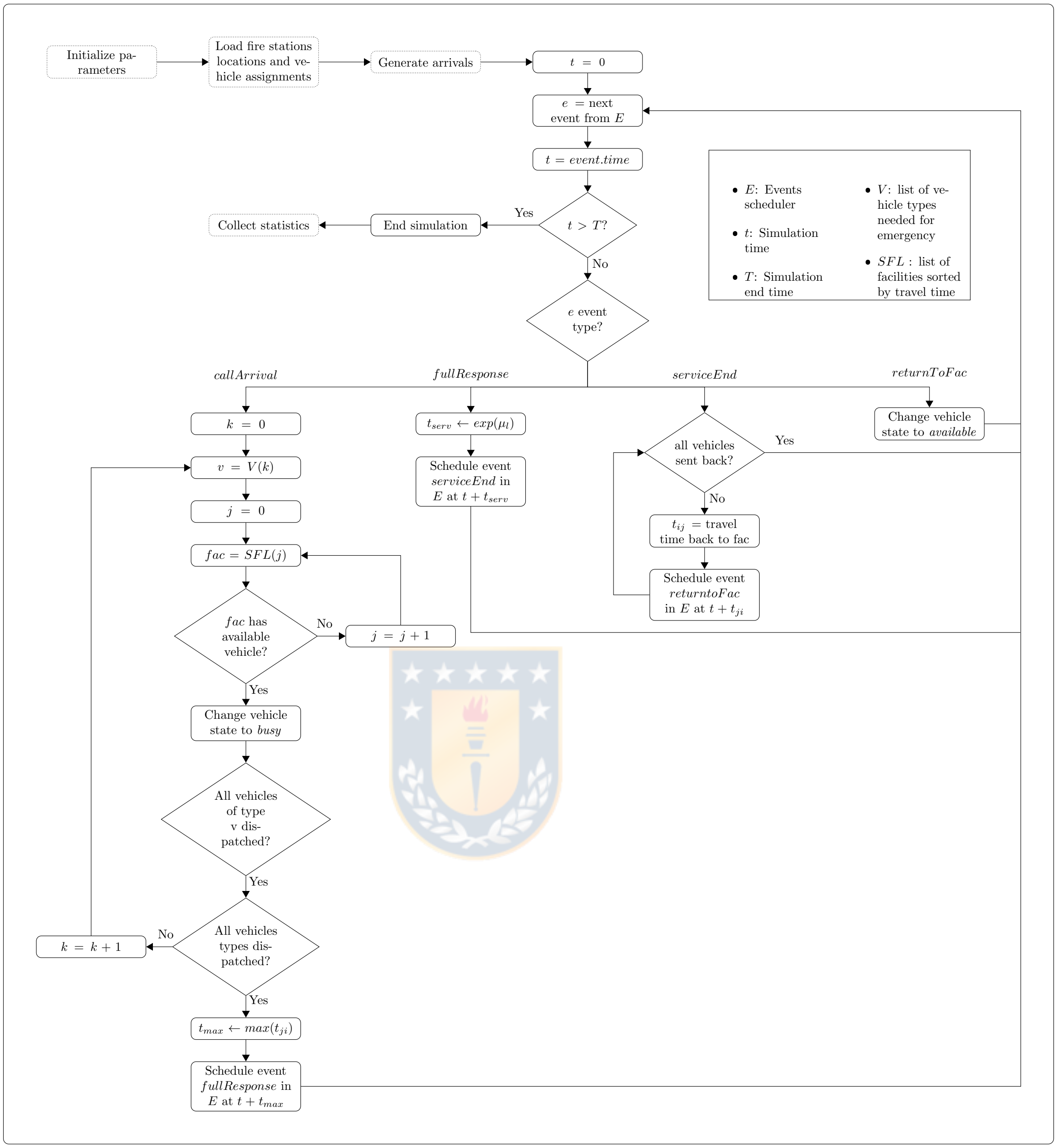
### 3.4.1 Model description

We modeled attendance of emergencies by firefighters as a server-to-customer process as presented in Figure 3.2. For this study, we did not consider the time spent in transferring the alert from the emergency call center to each fire station. Moreover, when a call arrives, the closest available vehicles are sent following an emergency type-dependent dispatch policy. In real emergencies, vehicles start attending the event once they arrive at the scene, however, the response time depends on the arrival of all required vehicles; thus, the maximum response time from the dispatched vehicles is recorded as the actual time. Then, once the emergency is served all vehicles return to their respective fire station. The main outputs of this model are the average utilization per vehicle type for each fire department and the actual coverage of emergencies. Finally, the structure of the program is presented in appendix B.

## 3.5 Spatio-temporal sampling

When simulating emergency service operations, an adequate event arrival process is essential to maintain face validity. Moreover, when facing location problems, we not only need to consider the arrival rate of emergencies, but also the places where they occur. Both temporal and spatial components are required to properly describe the emergencies' arrival process and must be considered when modeling it. The first sampling method that a modeler might try would be to use available data to recreate past scenarios and obtain the system's performance for each given location decision. The benefit of this method lies in

**Figure 3.2:** Simulation Flow chart.




the simplicity of using real records to recreate the arrival process in the simulation model. However, if the number of observations is not enough to properly evaluate the system, the modeler might be stuck by not being able to obtain valid results, even though he built an appropriate simulation model. Another option to prevent this problem would be to adjust statistical distributions using available data, and samples from them to model the arrival process. This method not only enhances the reproducibility of different scenarios, but also the ability to test location decision on a greater number of replications to report results that are statistically significant. Although this method allows the modeler to have an infinite number of samples, the adjustment of a statistical distribution is highly dependent on the quality of the original data. Because of this, precautions must be taken to avoid misrepresentation of the emergency arrival process which could lead to incorrect performance results. Thus, the main challenge faced when geographically sampling data is to couple both temporal and spatial components in a way that the entire process represents the real behavior of emergency occurrence. Hence, we loosely couple two modern approaches that individually reproduce the temporal and spatial demands by simulating the emergency arrival process using the NSNR arrival process for the temporal component, and a KDE for the spatial component.

### 3.5.1 Temporal

Usually when modeling emergency calls arrival processes, a Poisson distribution is assumed due to its statistical properties. Although this is a good approximation, it assumes independence between arrivals and a coefficient of variation of interarrivals equal to 1. In reality, arrival processes generally deviate from this statistical distribution and in most cases there is correlation between arrivals. Moreover, Whitt (2007) stated that results when simulating queueing models of service systems with non-homogeneous arrival processes may be misleading when assuming a constant arrival rate. Nelson and Gerhardt (2011) proposed a method to simulate a sequence of interarrival times $\{W_n, n \geq 1\}$ such that the arrival counting process $I(t) = \max\{n \geq 0 : V_n \leq t\}$ (where $V_n = \sum_{i=1}^{n} W_i$) is non-stationary and non-renewal (NSNR) by specifying a time-varying rate $r(t)$, an estimate squared coefficient of variation $cv^2$, and lag-j autocorrelation (using $j = 1$) of the base process. The authors start by defining a set of stationary non-negative interarrival times $\{X_n, n \geq 1\}$, being $S_n$ the time of the $n$th arrival (considering $S_0 = 0$ and $S_n = \sum_{i=1}^{n} X_i$, for $n = 1, 2, \ldots$), and $N(t)$ the number of arrivals that have occurred on or before time $t$, with $N(t) = \max n \geq 0 : S_n \leq t\}$. Additionally, they assumed that $N(t)$ is initialized in equilibrium. For this process, Sriram and Whitt (1986) define the index of dispersion for counts ($IDC$) as (note that $IDC = 1$ for a Poisson process, and $IDC = cv^2$ for an equilibrium renewal process):

$$IDC = \lim_{t \to \infty} \frac{Var\{N(t)\}}{E\{N(t)\}} \quad (3.26)$$

Then Gusella (1991) defines the index of dispersion of intervals ($IDI$), that is equal to de $IDC$ for most stationary arrival processes, as:

$$IDC = IDI \equiv \lim_{n \to \infty} \frac{Var\{S_n\}}{nE^2\{X_2\}} \quad (3.27)$$

$$= cv^2(1 + 2\sum_{j=1}^{\infty} \rho_j) \quad (3.28)$$

where $\rho_j$ is the lag-j autocorrelation of the stationary interarrival times $X_n$. As a result, $IDC$ takes into account variability (by including $cv^2$) and dependence (with $1+2\sum_{j=1}^{\infty} \rho_j$) in a stationary arrival process.

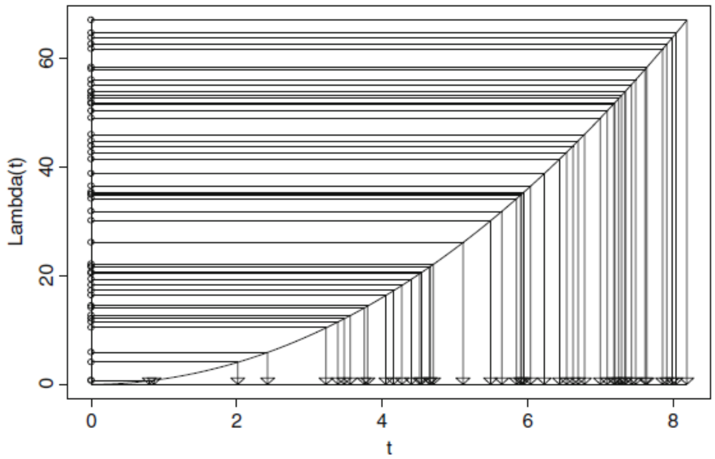
On the other hand, the authors generalize an algorithm (previously proposed on Gerhardt and L. Nelson (2009)) for generating NSNR processes. Define $r(t), t \geq 0$ as the integrable non-negative arrival rate for $I(t)$ and let $R(t) = \int_0^t r(s)ds$. Therefore, the arrival rate $r(t)$ is the instantaneous rate of change of the number of arrivals of non-stationary arrival process $I(t)$ at time $t$. For $s \in \mathbb{R}^+$, define $R^{-1}(s) \equiv inf\{t : R(t) \geq s\}$. The algorithm is as follows:

- Step 1: Set $V_0 = 0$, index counter $n = 1$. Generate $S_1$. Set $V_1 = R^{-1}(S_1)$.
- Step 2: Return interarrival time $W_n = V_n - V_{n-1}$.
- Step 3: Set $n = n + 1$. Generate $X_n$. Set $S_n = S_n - 1 + X_n$ and $V_n = R^{-1}(S_n)$.
- Step 4: Go to Step 2.

The authors then prove that the resulting $I(t)$ has $E\{I(t)\} = R(t)$ for all $t \geq 0$ and $Var\{I(t)\} \approx IDC * R(t)$ for large $t$. In conclusion, the inversion method maintains the arrival rate and transfers the IDC of the base process to the NSNR arrival process. An example for the inversion method is shown in Figure 3.3

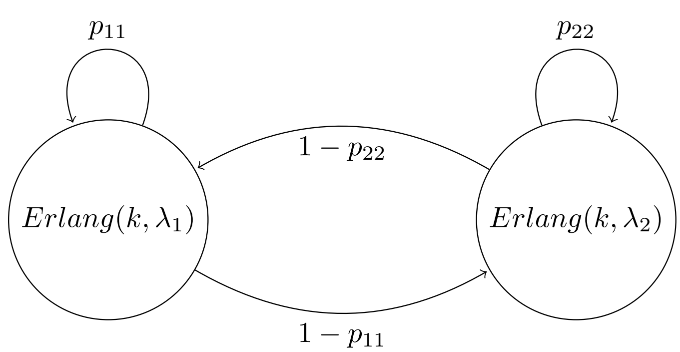
For the base arrival process $N(t)$, the authors propose a Markov Mixture of Erlangs of Common Order (Markov-MECO) presented by Johnson (1998). The Markov-MECO is

**Figure 3.3:** Illustration of the inversion method when $\lambda(t) = 2t$. *From Nelson (2013)*




a special case of a Markovian arrival process that extends the Mixture of two Erlangs of Common Order (MECO) renewal process for non-renewal processes presented in Johnson and Taaffe (1989). First, a MECO is a phase-type distribution, which is defined as the probability distribution of the time until absorption of a Markov process with a finite number of transient states and one absorption state. The advantage of using this distribution lies in its convenience for moments matching, which allows replicating most renewal processes by specifying its first three moments. Based on this distribution, the Markov-MECO extends the MECO to non-renewal arrivals by controlling dependence between interarrival times using a Markov process as shown in Figure 3.4. Here, the transitions probability determine from which Erlang the next interarrival time is generated, and therefore the generation of an interarrival time depends on the state from which the previous interarrival time was generated.

**Figure 3.4:** Markov-MECO process.



The Markov-MECO is used to generate $X_n$ in the algorithm shown above. Since the arrival rate for the base Markov-MECO must be 1 (therefore it's mean is 1), three additional parameters must be calculated: the coefficient of variance ($cv$), the skewness, and some measure of dependence between interarrival times. For the skewness, they select a Markovian distribution that is fully specified by knowing only its mean and $cv$. Because interarrival times are frequently more variable ($cv > 1$) or more regular ($cv < 1$) than Poisson, we have that:

- If $cv < 1$, then we use a MECon distribution (Mixture of Earlangs of consecutive order (Tijms (1994)) and extract its implied third moment.
- If $cv \geq 1$, then we use a balanced hyper-exponential distribution Sauer and Chandy (1975) and extract its implied third moment.

Finally, the dependence parameter between interarrival times can be specified by $\rho_1$ (autocorrelation lag-1).

### 3.5.2 Spatial

In most cases where geographic data is used, points are aggregated into suitable geographic units to model spatial occurrence. For example, square grids are used to aggregate data inside of each quadrant to generate a centroid that acts as a single demand point from which travel times are computed. Although this decreases the number of calculations that have to be made, and therefore computational time, aggregation adds an inevitable distortion to real locations, affecting travel times. Despite this, from the generated square grid, we can calculate a histogram for latitude and longitude variables to have a glance at the spatial distribution across each axis. This method might seem the right way of approximating a spatial density, but as we said before, the resulting discrete grid may distort points distribution due to the sensibility associated with the grid's size. Excessive aggregation (bigger squares) will spread data over a larger area, and smaller square sizes will not show spatial trends. To avoid this issue, a smoother density estimator can be used. A kernel density estimator (KDE), as defined in Chacón and Duong (2018), has the following equation:

$$\hat{f}(x; H) = n^{-1} \sum_{i=1}^{n} K_H(x - X_i) \tag{3.29}$$

Where K is an integrable function with unit integral, $\hat{f}$ is the smoothed probability mass from a data point $X_i$ in the local neighborhood, according to the scaled kernel, to represent the unobserved data point $x$. Moreover, $H$ is the bandwidth tuning parameter which controls orientation and the extent of the smoothing applied via the scaled kernel $K_H(x) = |H|^{-1/2} K(H^{-1/2} x)$, where $|H|$ is the determinant of $H$ and $H^{-1/2}$ is the inverse of its matrix square root. A scaled kernel is positioned so that its mode coincides with each data point $X_i$, which is expressed mathematically as $K_H(x - X_i)$. To ensure that the probability mass of $\hat{f}$ remains one, the scaled kernels are summed and divided by $n$. Kernels are placed on each data point, and the entire kernel density estimator inherits the smoothness of the individual kernels. The most used multivariate kernel function is the normal kernel:

$$K(x) = (2\pi)^{-d/2} \exp(-\frac{1}{2} x^T x) \tag{3.30}$$

Which is the standard $d$-variate normal density function. The scaled, translated normal kernel is:

$$K_H(x - X_i) = (2\pi)^{-d/2} |H|^{-1/2} \exp\{-\frac{1}{2}(x - X_i)^T H^{-1} (x - X_i)\} \tag{3.31}$$

Which is a normal density centered at $X_i$ and with variance matrix H. For simplicity, a single variance bandwidth (circular) kernel is used in this study. Additionally, because the purpose of adjusting a KDE is the forecast of events, the machine learning method *cross validation* is applied to obtain the optimal bandwidth parameter. This method uses a training set (a subset from the available data) to evaluate the fitness on the prediction of a KDE by using a performance metric (log probability density for the KDE). Also, more than one training set can be used from the original dataset in order to decrease variation by computing the average of the performance metric. Thus, a set of bandwidths parameters are tested and the best is used to adjust the KDE for the spatial distribution.

### 3.5.3 Sampling procedure

Once the temporal and spatial components have been modeled, the following sampling procedure is done to generate the arrivals of events. Considering the nature of emergencies, event types are assumed to be independent between them and the sampling procedure is done separately for each one. First, in order to couple both components of the events' arrival process, the spatial distribution of events is assumed to be dependent on the seasonal behavior of emergencies. This is a rather fair assumption when analyzing emergencies occurrence due to their correlation with natural variables associated. Thus, a KDE is used to adjust the spatial distribution per season for each emergency type. On the other hand, the available data is categorized into the desired time unit (hours, shifts, days) for each season in order to fit the temporal distribution. In this study, arrival rates vary on a daily basis showing variable intra-week intensity, therefore the temporal component of observations is fitted using the NSNR arrival process discussed above, considering each day of the week and using the available data for each season. Finally, the interarrival times are generated following a previously determined simulation horizon and the geographic location for each event is sampled from the corresponding KDE, as shown in **??**.

# Chapter 4

# Case Study

The proposed iterative method was applied to the twelve fire departments that operate in the Concepción Province, Chile, shown in Figure 4.1. First of all, concerning emergency attendance, each fire department serves a delimited area associated with a district (or commune). These districts are categorized based on the amount of population they have: i) *Suburban* $< 5.000$ people, ii) *Mesourban* $\geq 5.000$ and $< 40.000$ people, and iii) *Urban* $> 40.000$ people. Moreover, fire departments are composed of several fire stations, which are managed by their group of volunteers and vehicles. Basic and specialty vehicles are assigned to each fire station, taking into account the volunteers' experience in serving different types of emergencies, which are classified into the following categories: i) *Fire*: urban fires, ii) *Rescue*: people's rescues mainly due to car accidents, iii) *Forest*: forest fires near urban areas, iv) *Hazmat*: hazardous materials related accidents. As for vehicle types, a basic vehicle *(B)* is defined as a vehicle equipped with a pump engine that can attack any fire-related emergency. Additionally, specialty vehicles are categorized according to the type of emergency they serve: i) *Ladder truck (L)* for fire emergencies, ii) *Rescue truck (R)*, iii) *Hazmat truck (H)* and iv) *Forest truck (F)*. When an emergency occurs, vehicle dispatch is made by following the region-dependent dispatch policy presented in Table 4.2, where each type of emergency requires a specific combination of basic and specialty vehicles. In order to consider an emergency to be successfully attended, all vehicles specified in the dispatch policy must arrive under a certain response time target. In case backup is needed, vehicles from other fire departments can be dispatch.

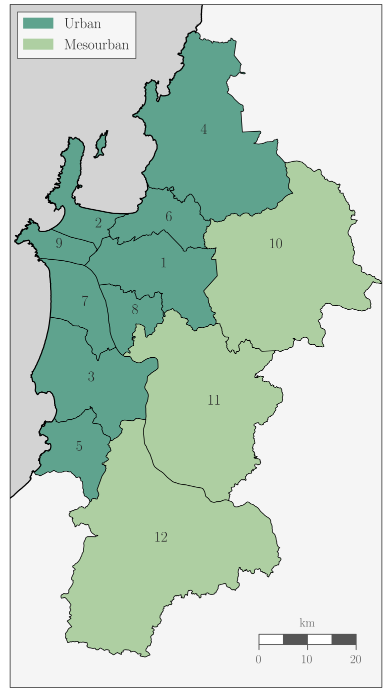
**Table 4.1:** Districts information.

| | Districts | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
| Fire stations | 10 | 10 | 8 | 7 | 4 | 4 | 4 | 3 | 2 | 3 | 2 | 2 | 58 |
| B | 14 | 12 | 6 | 6 | 4 | 6 | 3 | 5 | 5 | 4 | 2 | 3 | 70 |
| Q | 5 | 1 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 0 | 11 |
| R | 4 | 6 | 2 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 0 | 22 |
| F | 3 | 4 | 2 | 2 | 0 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 16 |
| H | 2 | 3 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 9 |

**Table 4.2:** Vehicles dispatch policy.

| *Emergency type* | *Avg. serv. time (hrs.)* | *Vehicles dispatch* | | |
|---|---|---|---|---|
| | | Urban | Mesourban | Suburban |
| Fire | 1.50 | 2B & 1L | 1B & 1L | 2B |
| Rescue | 0.75 | 1B & 1R | 1B & 1R | 1B & 1R |
| Hazmat | 3.50 | 1B & 1H | 1B & 1H | 1B |
| Forest fire | 3.00 | 1B & 1F | 1B & 1F | 1F |

**Figure 4.1:** Concepcion province




## 4.1 Dataset

The dataset used in this study consists of 18.066 emergencies that occurred during 2016 and 2017 in the Concepción Province, Chile. Of these data points, 7.286 events had errors on their geographic information while maintaining a correct registry on when they occur. Thus, in order to correctly adjust the temporal and spatial distributions of events, this subset is only used for the modeling of the NSNR arrival process and is discarded for the generation of the KDE. Additionally, there is no registry of events on district 2 for 2016. Moreover, due to the specific nature of each type of emergency registered, we consider in the following analysis that they are independently distributed from each other. Then, the dataset was used to simulate the emergency arrival process and evaluate the average utilization of vehicles for each Fire Department. Furthermore, the available dataset is used to define the demand parameter $d_{il}$. Besides, considering the candidate nodes for the location of fire stations, we consider the 57 current fire stations, and their vehicles present, in the region of study as available for relocation. Additionally, 238 nodes were selected from the centroids of squares on a grid of 1000m x 1000m were no fire stations currently exist and used as candidates nodes to locate new facilities.

## 4.2 DES model

The simulation model presented in Section 3.4 was implemented to compute the average utilization parameters $\rho_{kn}$. This model was developed using the simulation library *SimPy*, which is entirely based on Python. A number of 30 replicas were generated based on 6 years of events with 5 different traffic scenarios each. Specifically, four equally distributed levels of speed variation (+4%, 0%, -4%, -8%) were determined to alter the speed of each street segment when computing the travel time from each fire station to an emergency. Finally, a temporal and spatial analysis was developed for the above-mentioned dataset in order to compute the parameters needed for the loosely coupled spatio-temporal sampling procedure, which is described in the following sections.

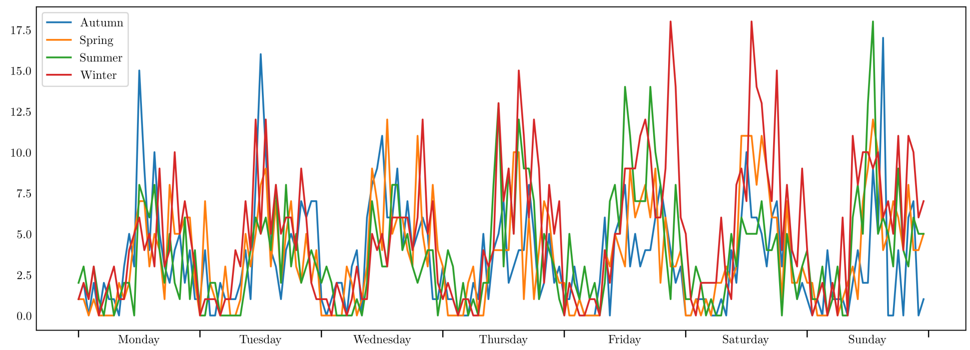
### 4.2.1 Temporal analysis

When analyzing the dataset, a variation on arrival rates occurs on the following temporal levels: seasonal (summer, winter, spring, autumn), per weekday (Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday), and hourly (24 hrs). In Figure 4.2, the weekly frequency for each season is presented for fire and forest emergencies. It can be seen in Figure 4.2(a) a higher count of fire events occur during Winter compared with other seasons. This behavior is explained by the use of combustion stoves in most homes during cold seasons, which increases the risk of a fire-related emergency in residential areas. On the other hand, forest emergencies have a specific seasonal behavior having their frequency peak during Summer, as shown in Figure 4.2(b). These findings agree with the firefighters' experience of these phenomena. Thereby, the available dataset was divided into categories considering season and weekday (e.g. Summer-Monday, Winter-Monday, Winter-Tuesday, and so on). Then, the hourly arrival rate was obtained considering each registered date as an observation for its corresponding category. For example, 22 Fridays were observed during summertime. Therefore, the same number of observations was obtained from the dataset for each hour of the day for that category.

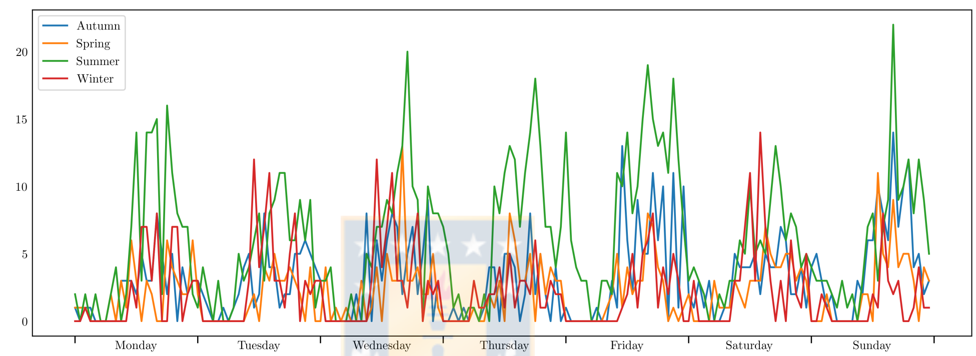
The NSNR arrival process needs three input parameters calculated from the emergency interarrival times: the coefficient of variation ($cv$), $lag-1$ correlation ($\rho_1$) and a time-varying arrival rate ($r(t)$). First, Gerhardt and L. Nelson (2009) determined that for a

**Figure 4.2:** Weekly frequency of emergencies.

**(a)** Fire emergencies




**(b)** Forest emergencies




non homogeneous arrival process $N(t)$ generated by a customized inversion method, $cv$ can be estimated by:

$$\frac{Var(N(t))}{E(N(t))} \approx \hat{\sigma}^2 \quad \text{for large t} \tag{4.1}$$

Then, on Nelson (2013), a method is presented to estimate a measure of deviation, for a particular arrival process, from being a Poisson process. An example of this procedure is presented in Table 4.3 for 22 consecutive Fridays. For each of the categories presented above, a table was built registering each observation by the hourly occurrence of emergencies. Next, the cumulative count of realizations ($C_j(t), j = 1, 2, \ldots, 22$ and $t = 0, 1, \ldots, 23$) is calculated for each observation. After this, the variance of the number of arrivals by time $t$ is estimated by Equation 4.2:

$$V(t) = \frac{1}{k-1}\sum_{j=1}^{k}(C_j(t) - \hat{\Lambda}(t))^2 \tag{4.2}$$

**Table 4.3:** Example table for $\hat{\sigma}$ estimation based on observations of fire emergencies occurred on Friday during Summer

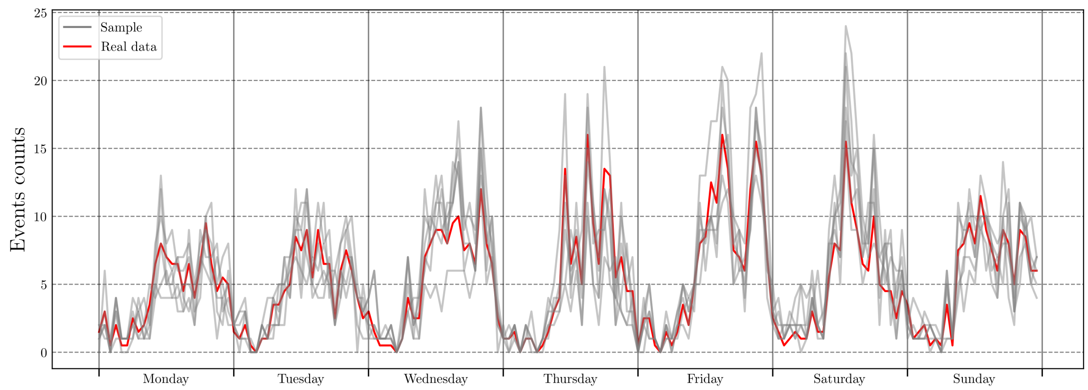
| $t$ | # | $C_1(t)$ | # | $C_2(t)$ | ... | # | $C_{22}(t)$ | $\hat{\Lambda}(t)$ | $V(t)$ | $\frac{V(t)}{\hat{\Lambda}(t)}$ | $r(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | ... | 0 | 0 | 0.19 | 0.16 | 0.85 | 0.19 |
| 1 | 0 | 1 | 0 | 0 | ... | 0 | 0 | 0.33 | 0.38 | 1.15 | 0.15 |
| 2 | 1 | 2 | 0 | 0 | ... | 1 | 1 | 0.44 | 0.49 | 1.10 | 0.11 |
| 3 | 0 | 2 | 0 | 0 | ... | 0 | 1 | 0.52 | 0.49 | 0.95 | 0.07 |
| 4 | 0 | 2 | 0 | 0 | ... | 0 | 1 | 0.59 | 0.71 | 1.20 | 0.07 |
| 5 | 0 | 2 | 0 | 0 | ... | 0 | 1 | 0.63 | 0.70 | 1.12 | 0.04 |
| 6 | 0 | 2 | 0 | 0 | ... | 0 | 1 | 0.78 | 0.79 | 1.02 | 0.15 |
| 7 | 0 | 2 | 0 | 0 | ... | 0 | 1 | 0.85 | 0.98 | 1.15 | 0.07 |
| 8 | 0 | 2 | 1 | 1 | ... | 0 | 1 | 1.04 | 1.04 | 1.00 | 0.19 |
| 9 | 0 | 2 | 3 | 4 | ... | 1 | 2 | 1.41 | 2.40 | 1.71 | 0.37 |
| 10 | 0 | 2 | 0 | 4 | ... | 0 | 2 | 1.78 | 2.95 | 1.66 | 0.37 |
| 11 | 0 | 2 | 1 | 5 | ... | 0 | 2 | 2.11 | 4.18 | 1.98 | 0.33 |
| 12 | 0 | 2 | 0 | 5 | ... | 0 | 2 | 2.78 | 6.87 | 2.47 | 0.67 |
| 13 | 0 | 2 | 0 | 5 | ... | 0 | 2 | 3.30 | 10.83 | 3.29 | 0.52 |
| 14 | 1 | 3 | 0 | 5 | ... | 1 | 3 | 3.74 | 12.81 | 3.43 | 0.44 |
| 15 | 0 | 3 | 0 | 5 | ... | 0 | 3 | 4.33 | 17.85 | 4.12 | 0.59 |
| 16 | 0 | 3 | 0 | 5 | ... | 0 | 3 | 4.85 | 19.67 | 4.05 | 0.52 |
| 17 | 0 | 3 | 2 | 7 | ... | 1 | 4 | 5.63 | 26.40 | 4.69 | 0.78 |
| 18 | 1 | 4 | 0 | 7 | ... | 0 | 4 | 6.11 | 32.72 | 5.35 | 0.48 |
| 19 | 0 | 4 | 2 | 9 | ... | 1 | 5 | 6.52 | 34.72 | 5.33 | 0.41 |
| 20 | 1 | 5 | 0 | 9 | ... | 0 | 5 | 6.89 | 39.10 | 5.68 | 0.37 |
| 21 | 0 | 5 | 0 | 9 | ... | 0 | 5 | 7.07 | 40.15 | 5.68 | 0.19 |
| 22 | 1 | 6 | 0 | 9 | ... | 1 | 6 | 7.48 | 39.26 | 5.25 | 0.41 |
| 23 | 0 | 6 | 1 | 10 | ... | 0 | 6 | 7.74 | 44.20 | 5.71 | 0.26 |

where $\hat{\Lambda}(t)$ is the average cumulative count for time period $t$. Finally, an estimator for $\hat{\sigma}^2$ is computed using Equation 4.3:

$$\hat{\sigma}^2 = \frac{1}{m} \sum_{i=1}^{m} \frac{V(t_i)}{\hat{\Lambda}(t_i)} \tag{4.3}$$

Additionally, the time-varying arrival rate $r(t)$ is also calculated from Table 4.3 by taking the average of the arrival rates per hour. On the other hand, the $lag - 1$ correlation is computed from the observed interarrival times. Once these parameters are calculated for each data category, we use the NSNR arrival process to sample emergency interarrival times for each emergency type. Then, because each category corresponds to a particular season and weekday, we sample emergencies following a simulation time horizon determined by the planner, e.g. when sampling one year of events, 2020 is chosen as a reference year to determine which weekday corresponds to a specific date (season is inherently associated to each date); therefore, the 365 days are sampled by using the distribution adjusted to their resulting category. Figure 4.3 shows the realizations drawn from the proposed distribution of the arrival of fire emergencies during winter.

Figure 4.3: Comparison between real data and sampled arrivals for fire emergencies during winter




### 4.2.2 Spatial analysis

For the spatial component of the emergency arrival process, a KDE was adjusted for each emergency type. As mentioned in Section 3.5.3, to couple this sampling with the temporal component, we grouped the available data into seasonal categories and calculated a KDE for each set of events using the open-source library *Scikit-Learn* with a one-dimension bandwidth kernel specified by the user. As shown in Figure 4.4, KDEs are sensible to

**Figure 4.4:** KDEs bandwidths comparison

**(a)** Bandwidth 1: $3x10^{-2}$. **(b)** Optimal: $3.894x10^{-3}$. **(c)** Bandwidth 2: $1.5x10^{-3}$.

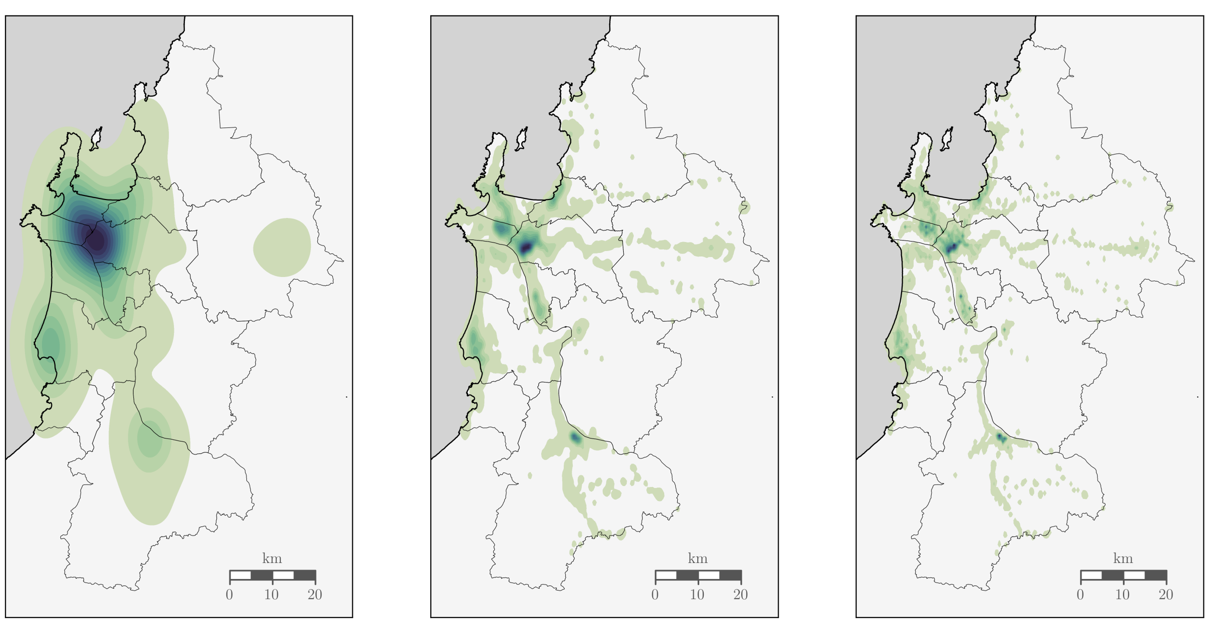
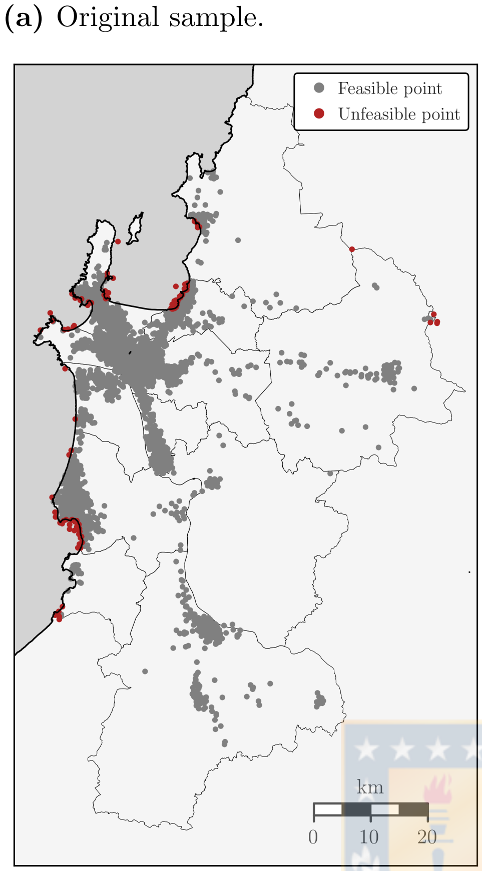
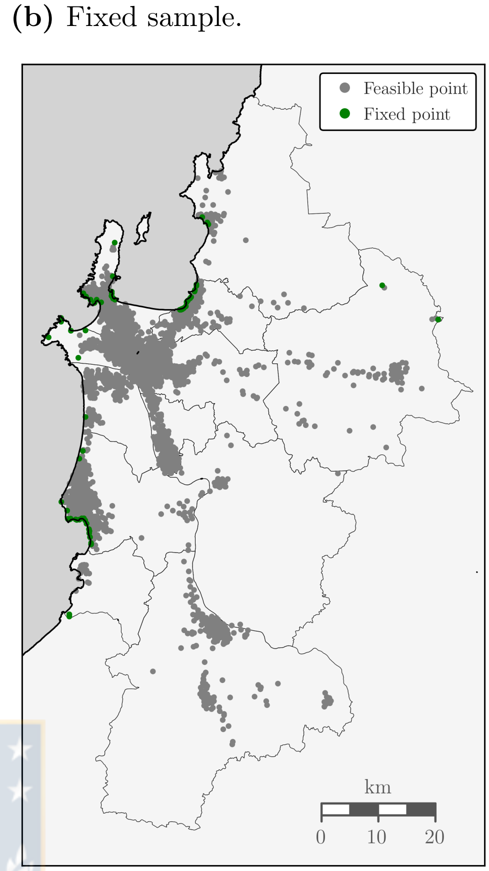


**Figure 4.5:** KDE with sampled data outside the region with relocated points.




variations on bandwidths values, thus, a set of parameters are tested using k-fold cross-validation to choose the best fit of the spatial distribution. The objective of this method is to evaluate the performance of a predicting model (in our case a KDE) by testing a set of candidates parameters (bandwidths values). First, the original data is divided into $k$ equal-size subsets. One of them is selected as the validation set to test the model, whereas the remaining subsets are used to train the predictive model. This procedure is repeated $k$ times, for a fixed set of parameters, using each subset as a validation set, and then the average evaluation metric is computed. Once all candidate parameters are tested, the one with the best performance is selected to adjust a KDE to sample the geographic locations of events.

Although KDE depends on the available data points, due to geographic constraints, a problem arises: sampled points may appear on unfeasible areas like lakes, rivers or even outside of the study region. A first simple solution is to eliminate these points and only use those that are sampled in feasible areas. However, by doing this the probability mass of the KDE is altered by conditioning those regions where points are eliminated. An alternative to circumvent this problem is to ´fix' unfeasible points by moving them inside the feasible region. Two options were considered as new locations for unfeasible points: the first one is the closest point on the boundary of the study region, and the second one is the closest feasible sampled point. For computation efficiency, the second option was selected. Figure 4.5(a) illustrates this process presenting a sample of events directly obtained from the KDE, whereas Figure 4.5(b) shows the sampling after the fix.

### 4.2.3 Response time

Generally, the response time of emergencies is composed of three intervals: the time on the emergency call center, the turnout time and the travel time. The first one describes the time spent from the moment an emergency call arrives until an alarm is set on the fire station chosen to attend the event. The second time interval considers the time when the alarm is set until the selected vehicle exits the fire station. Finally, the travel time consists of the time it takes to the vehicle to arrive at the scene of the emergency. For the purpose of this study, we did not consider the time on the call center as part of the response time. Because currently, Chilean firefighters do not have a standard response time goal for the different types of emergencies, we developed an emergency coverage target based on the National Fire Protection Association standards. This entity is responsible for determining the standard norms for firefighters' operations in the US and delivers guidelines for their compliance. Specifically, they developed the NFPA-1720 for volunteer fire departments which, among with the response time data available of Chilean firefighters, were used to determine the following goal for the response time of the entire fleet of vehicles dispatched following the policy in Table 4.2.

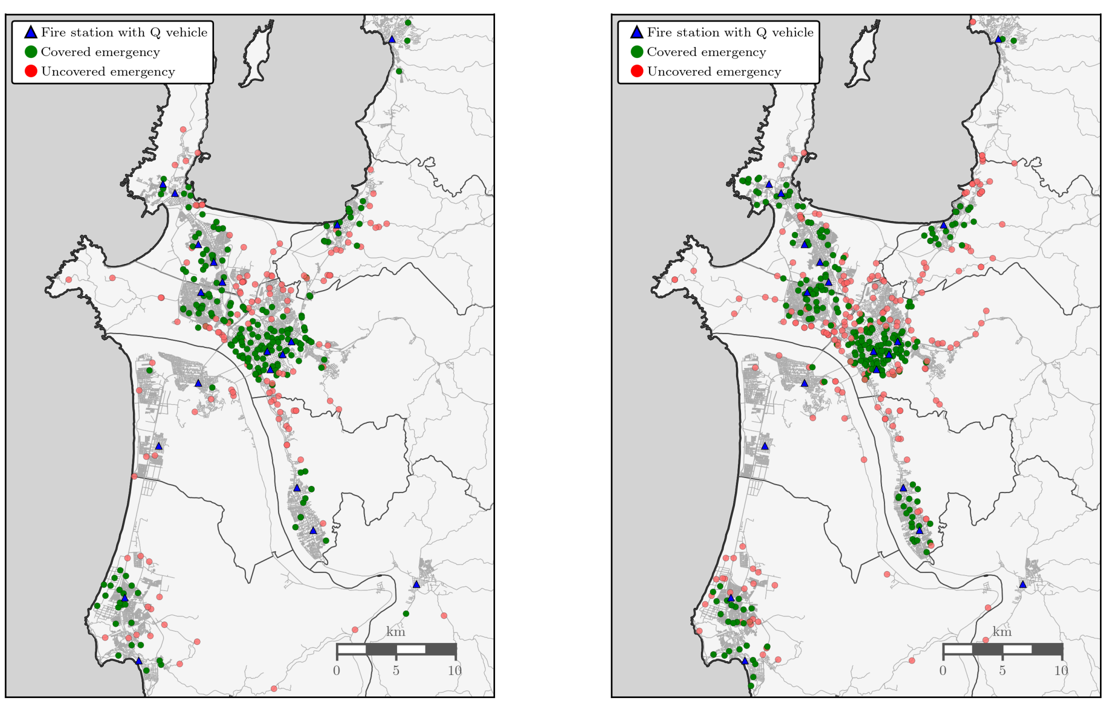
- 10 minutes for 90% of emergencies on urban districts.
- 12 minutes for 80% of emergencies on mesourban districs.
- 20 minutes for 80% of emergencies on suburban districts

Due to lack of information, the turnout time was modeled by adjusting a triangular distribution with a lower bound of two minutes, a mode of four, and an upper bound of six. These values were obtained by interviewing volunteers of different fire departments, and considering that some fire stations on mesourban and suburban districts do not always have an available crew on site, which increases the turnout time. For estimating travel time for both simulation and MIP models, we used the current street network and observed vehicle traffic of the Concepcion province. First, we built a street network graph using Open Street Maps (OSM) data, maintaining the OSM street classification based on infrastructure and traffic volume: i) *motorway*: highways, ii) *trunk*: alternative highways and roads, iii) *primary*: urban streets, iv) *secondary*: residential streets, v) *terciary*: small residential streets. Then, in order to model actual traffic behavior, we sampled 40 roads which in our experience are considered representative of the street network and collected their reported speed from Google Maps between January and May of 2019. Furthermore, we determined the following time schedules based on traffic's peak and regular hours to sample the data: *(07:00-09:00)*, *(09:00-12:00)*, *(12:00-14:00)*, *(14:00-17:00)*, *(17:00-19:00)*, *(19:00-23:00)*, *(23:00-07:00)*. As a result, 70 days were sampled and the computed average speed for

**Figure 4.6:** Rescue emergencies coverage comparison between off-peak and peak hours.

**(a)** Off-peak hours: 22:00 - 07:00.

**(b)** Peak hours: 17:00 - 20:00.

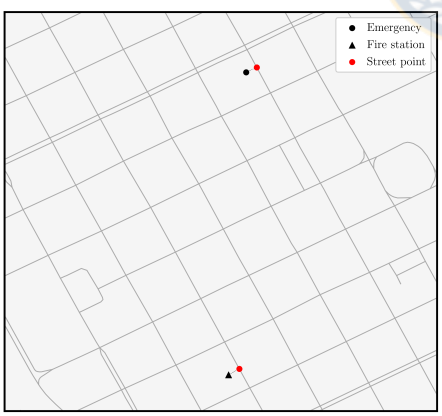


each street type was assigned to each street edge for the corresponding time schedule. To illustrate the difference in coverage due to time-varying speeds on a daily basis, Figure 4.6 of rescue emergencies during off-peak and peak hours.

Once the street network was built, including the estimated travel speed, we identified the closest street for each fire station and emergency event in order to compute the shortest path between both points considering travel time. Because we need a target point instead of a line to calculate the shortest path, two alternatives are presented: use either the start or end node of the closest street, or interpolate the actual closest point within the closest street edge. For the first case, we must consider that misrepresentation of the actual travel time might occur for points near long streets (mainly highways); thus, it seems more accurate to not only finding the closest street to each coordinate, but also the actual closest point within the street. An example is presented in Figure 4.7, where an event point is interpolated into its closest street.

**Figure 4.7:** Travel path from fire station to emergency.

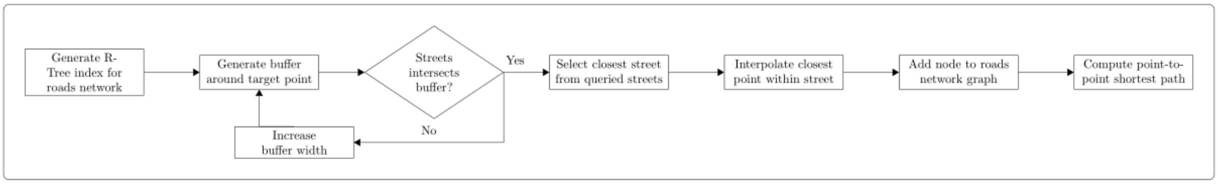
**(a)** Interpolation of event point on closest street.




**(b)** Shortest path from fire station to emergency.

Hence, we developed a computationally efficient method to compute travel times from point-to-point by using the closest street point in the roads network to both the fire station and the emergency, which is shown in Figure 4.8. First, an R-Tree is used to facilitate the search of nearby streets. An R-Tree us a multidimensional tree data structure commonly used to access geographic data, that uses multi-level bounding rectangles (which are considered as leaves of the tree) to classify geographic objects. The tree structure is

**Figure 4.8:** Point-to-point travel time calculation procedure




built from higher-level bounding rectangles (which are considered parent nodes) that contain lower-level ones (siblings), recursively repeating this structure for the desired depth. This method takes advantage of the property that, when doing a spatial search to find the closest object to a polygon, if this does not intersect a certain bounding rectangle, then the closest object is not contained within that subtree. Next, the search continues along the remaining subtrees following the same logic. Once the R-Tree has indexed the entire road network, a buffer is generated around the target point in order to query for the nearest roads. The buffer width is determined by considering an approximate average distance between the emergency point and the street segments, using as a reference a heavily dense urban area. This efficiently reduces the time spent on distance calculation by limiting the number of candidates objects. Additionally, if no street segment is found the buffer width is increased until a road segment is returned, which is especially useful in rural areas. Next, we calculate the distance from the target point to each of the resulting segments obtained from this query to determine the closest street. Finally, we interpolate the sampled point to the closest street segment, adding it as a node to the road network graph and use it as a start point (or end point) to compute the shortest travel time.

# Chapter 5

# Results

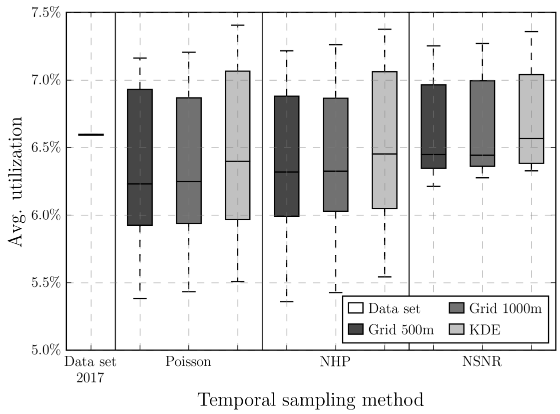
The following section presents an evaluation of the performance of both the FLEET-EXC model and its robust optimization counterpart. Moreover, they are compared with the discrete FLEET formulation to determine the effect the expected coverage has on the resulting solutions. The iterative procedure was implemented in Python programming language using the SimPy library for the simulation model, and commercial solver Gurobi's Python API to program and solve the MIP models. All experiments were solved using a computer equipped with an Intel i7-8550U processor and 16 GB of RAM.

## 5.1 Sensitivity Analysis

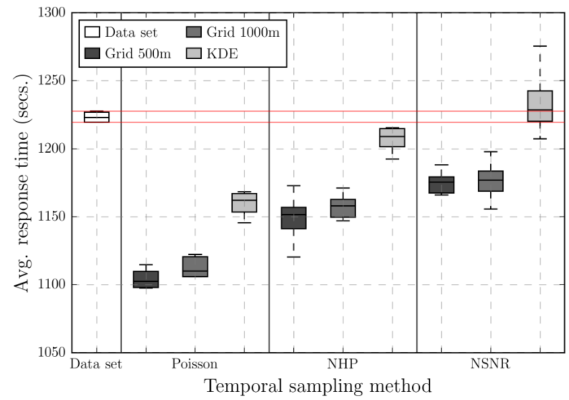
The proposed spatio-temporal sampling method is compared with those generally used in literature for the modeling of the emergency arrival process. Thus, we considered as alternatives to the NSNR arrival process, for modeling the temporal distribution, a Poisson process, and an NHP process; whereas for the modeling of the spatial distribution, we use two grids with quadrant sizes of 500 and 1000 meters respectively. These grids were used to model the distribution of events by aggregating the emergencies that occurred within a quadrant into a centroid that represents a weighted demand node with weight equal to the frequency of events. Figure 5.1 presents a comparison of the spatio-temporal sampling methods resulted from coupling the presented alternatives. Additionally, these sampling methods are compared with the results obtained from simulating the available dataset, and the response time is computed only for those events with verified geographic location. Finally, the average utilization is obtained from simulating the events from 2017.

**Figure 5.1:** Comparison of spatio-temporal sampling methods.

**(a)** Average utilization.




**(b)** Coverage.



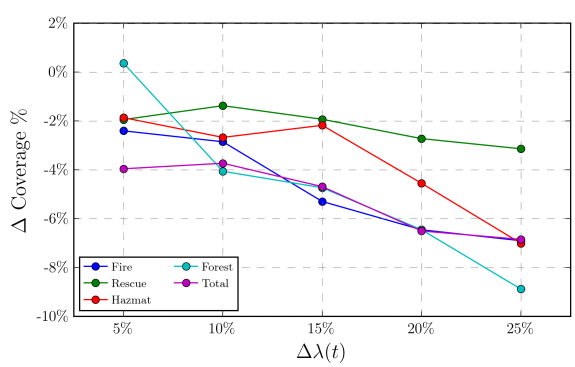

First, Figure 5.1(a) shows the simulated overall average utilization for each sampling method compared with the parameter simulated from the available dataset which has a mean of 6.6%. The results show that the NSNR arrival process median is closer to this parameter than both the Poisson and NHP processes, and has less variation towards a lower level of utilization. This is caused by the consideration of both the correlation between arrivals and the assumption that the arrival process is not Poisson, as shown in Nelson and Gerhardt (2011). Although the utilization of vehicles in firefighters systems is relatively low compared with other emergency systems such as ambulances, the Poisson and NHP processes underestimate the average utilization of vehicles. Furthermore, when comparing the spatial sampling methods when using NSNR, the median results from the KDE are closer to the dataset than those obtained using grids. Secondly, Figure 5.1(b) shows the difference in the simulated average response time. Considering the previous analysis, the Poisson and NHP processes have lower average response time due to less busyness of vehicles when compared with the NSNR process. Additionally, the KDE has higher average response times than the sampling from a grid, which is explained by the fact that grids simplify the computation of travel times into a single point, distorting the actual distance towards the set of events that happen inside that quadrant.

When comparing the simulated results from the dataset, the proposed spatio-temporal sampling method is the only one that achieves similar results. The variability shown for this sampling method is a result of the KDE's bandwidth that generates events more distant to streets than usual, and the NSNR arrival process correlation between events which increases the busyness of vehicles and, as a consequence, the response time. As a result, the combination of both methods can replicate the results obtained from the dataset, and adds a level of stochasticity to the generation of events. The NHP arrival process (the warhorse of most models in the literature) has similar behavior to the dataset, although it underestimates the response time by not considering the correlation of events. Finally, the combined use of both the NSNR arrival process and the KDE produces better modeling of the emergency arrival process with the available data.

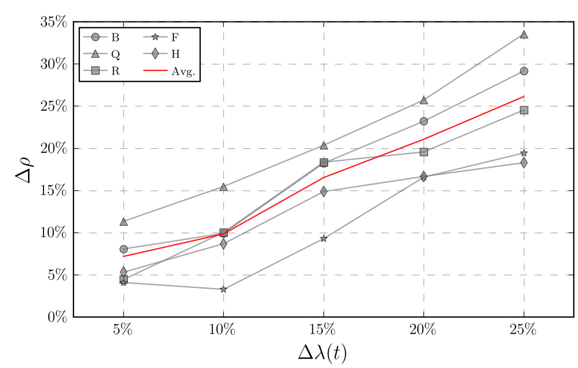
Then, we study the behavior of the expected coverage results for the FLEET-EXC model under an increase in the arrival rate of events $\lambda(t)$, presenting the variation on coverage and average utilization $\rho$ in Figure 5.2. The utilization parameters tend to increase as a consequence of a higher frequency of events, affecting both the resulting coverage and the resulting solution from the iterative procedure. Therefore, five scenarios of arrival rate increment were evaluated to observe possible changes in the optimal solution. First, we begin by analyzing the variation on coverage of the current layout of fire departments shown in Figure 5.2(a). Because emergencies attended by firefighters are less frequent than other emergency services like ambulances, the effect of small increments in the arrival rate intensity on the coverage of emergencies depends mainly on the location of those new events. For example, Figure 5.2(a) shows that the percentage change in coverage tends to decrease in a lower magnitude than the percentage change in the arrival rate.

**Figure 5.2:** Sensitivity analysis of the coverage and utilization parameters of the FLEET-EXC model for an increment on the emergencies arrival rate.

**(a)** Coverage.




**(b)** Average utilization.

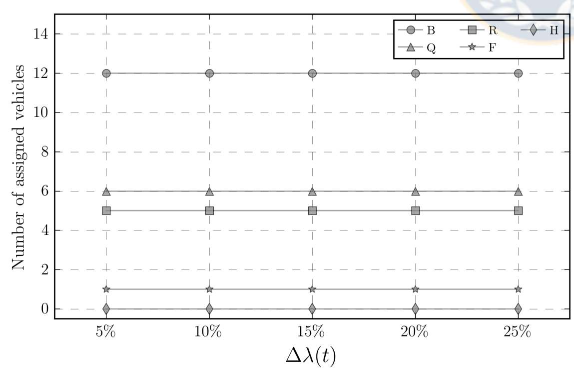


Moreover, when $\lambda(t)$ changes less than 10%, we observe a less than 5% percent change in the coverage of emergencies. That effect is mainly due to the proportion of events on current coverage areas. This is related to the increase in the average utilization shown in Figure 5.2(b), where $Q$ vehicles have the greatest increment on their average utilization.

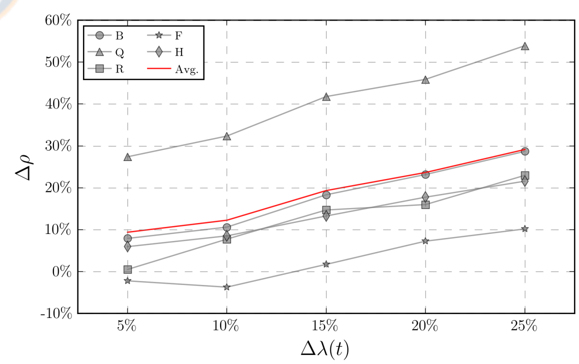
Figure 5.3 shows the effects of increasing the arrival rate intensity for the optimal results obtained using the FLEET-EXC model for the experiment where $p^n_{reloc} = p^n_{new} = 1 \quad \forall\, n \in N$ and $q = 1$. The results presented in Figure 5.3(a) show that the same solution is maintained between a 5% and 25% increase in $\lambda(t)$. Considering these results, it is fair to state that an increase of 25% on the arrival rate of events is the upper bound on a planning horizon of 5 years, which is within the scope of the problem presented in this study. Moreover, Figure 5.3(b) shows that the average utilization of $Q$ vehicles is the most sensitive to an increase in $\lambda(t)$, which may produce a preference for assigning these vehicles on higher intensity arrival rates.

**Figure 5.3:** Sensitivity analysis of vehicle assignment and utilization parameters for an increment on the emergencies arrival rate.

**(a)** Vehicles assignment.




**(b)** Average utilization.




## 5.2 Experimental results

In order to test different decision scenarios, a full factorial experiment was designed. As decisions parameters we considered: i) the number of new fire stations to locate ($p^{new}$), ii) the number of new basic and specialty vehicles assigned to each new facility ($q$), and iii) the number of fire stations relocations ($p^{reloc}$). Three levels were determined based on the feasibility of each decision, establishing a range from cero to two for all parameters.

Additionally, because $q > 0 \iff p^{new} > 0$, a subset of the full factorial experiments (where $(p^{new} > 0 \land q = 0) \lor (p^{new} = 0 \land q > 0)$) are unfeasible and not considered on the following analysis. Moreover, we assume that all fire departments share the same decisions parameters, with $p_n^{new} = p^{new}, p_n^{reloc} = p^{reloc}, \forall n \in N$. Furthermore, the experiment with $p_{new} = 0$, $p_{reloc} = 0$, and $q = 0$ is considered as the base case, because no changes are made to the current fire departments layout.

**Table 5.1:** Experiments results.

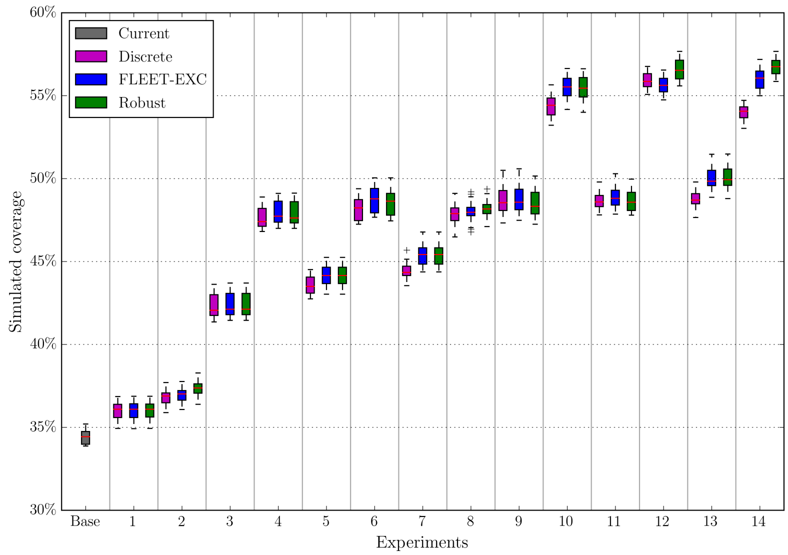
| Experiment | | | | Discrete model | | FLEET-EXC model | | | Robust model | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| # | $p_{new}$ | $p_{reloc}$ | $q$ | Runtime* | Avg. coverage | Runtime* | # iter. | Avg. coverage | Runtime* | # iter. | Avg. coverage |
| 1 | 0 | 1 | 0 | 616 | 36.01% | 1448 | 2 | 36.01% | 5100 | 7 | **36.02%** |
| 2 | 0 | 2 | 0 | 618 | 36.79% | 3441 | 6 | 36.94% | 3943 | 4 | **37.31%** |
| 3 | 1 | 0 | 1 | 643 | 42.27% | 1778 | 2 | **42.35%** | 2971 | 3 | **42.35%** |
| 4 | 1 | 0 | 2 | 637 | 47.65% | 1876 | 3 | **47.96%** | 3462 | 5 | 47.92% |
| 5 | 1 | 1 | 1 | 638 | 43.56% | 3683 | 4 | **44.12%** | 13635 | 4 | **44.12%** |
| 6 | 1 | 1 | 2 | 662 | 48.15% | 31495 | 9 | **48.74%** | 108850 | 8 | 48.5% |
| 7 | 1 | 2 | 1 | 649 | 44.39% | 7352 | 5 | **45.41%** | 58621 | 8 | **45.41%** |
| 8 | 1 | 2 | 2 | 4392 | 47.84% | 162655 | 14 | 47.98% | 587602 | 15 | **48.17%** |
| 9 | 2 | 0 | 1 | 810 | 48.72% | 2973 | 4 | **48.79%** | 2509 | 3 | 48.51% |
| 10 | 2 | 0 | 2 | 739 | 54.37% | 2672 | 4 | **55.53%** | 5416 | 7 | 55.48% |
| 11 | 2 | 1 | 1 | 1609 | 48.65% | 14319 | 12 | **48.86%** | 56869 | 10 | 48.65% |
| 12 | 2 | 1 | 2 | 761 | 55.91% | 10745 | 7 | 55.63% | 141023 | 10 | **56.56%** |
| 13 | 2 | 2 | 1 | 894 | 48.73% | 24242 | 7 | 49.96% | 215665 | 10 | **50.01%** |
| 14 | 2 | 2 | 2 | 824 | 54.00% | 49562 | 6 | 56.02% | 1141730 | 15 | **56.73%** |

* unit in seconds.

## 5.3 Coverage

Table 5.1 shows the main results of the MIP models for the proposed experiments, comparing the total runtime of the iterative procedure and the simulated average coverage of the obtained optimal solutions. As can be seen, even though the longest run took almost two weeks, most experiments were solved at an appropriate time the strategic planning nature of the problem. On the other hand, either the FLEET-EXC model or the Robust FLEET-EXC outperformed the Discrete model on every experiment, which translates on a better coverage when considering the utilization of vehicles to compute the expected coverage of emergencies. Furthermore, Figure 5.4 presents a more detailed comparison of the simulated coverage between models. The FLEET-EXC model has better performance than the Discrete model on every experiment except for experiment 12, while the Robust model has a lower coverage performance than the Discrete model only on experi-

**Figure 5.4:** Total coverage comparison between experiments.

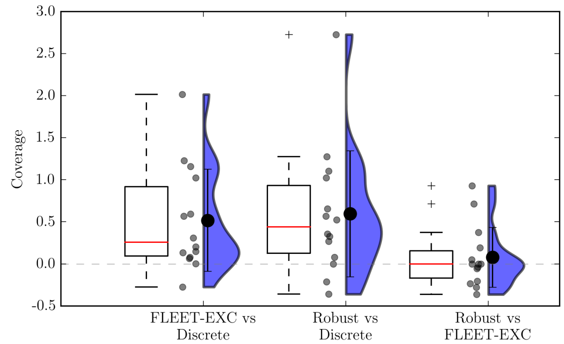



ment 9. When comparing the improvement in coverage with the base case, no experiment was able to achieve the proposed coverage target (90% of urban emergencies and 80% of mesourban ones). However, experiment 1 has a statistical difference in coverage with the current situation, stating that the smallest variation in the system evaluated will lead to an improvement in emergency coverage. Moreover, the Robust model has the highest emergency coverage when more changes are made on each fire department (experiments 12, 13 and 14), and when only relocation is allowed (experiments 1 and 2). On the contrary, the FLEET-EXC model has better performance when fewer relocations are made (experiments 4, 9 and 10). Finally, appendix C presents a comparison of coverage results between experiments per emergency type.

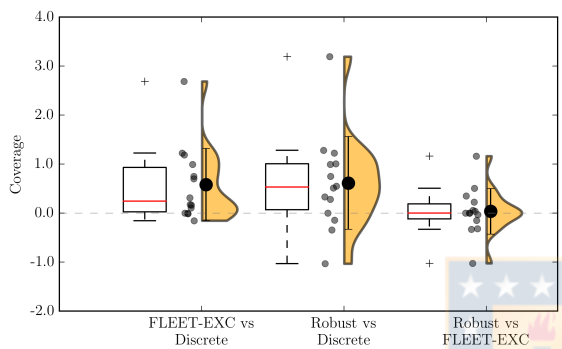
The experiments coverage mean difference between models is presented in Figure 5.5 for the various districts types. As stated before, the FLEET-EXC and Robust models have higher coverage than the Discrete model. Even though there is no statistical difference because the intervals contain zero, there is a clear practical difference because of the observed dispersion. Moreover, this difference is greater in mesourban districts than in urban ones, but the overestimation of coverage is greater in urban areas due to the higher utilization of vehicles. This may be explained by a lower level of coverage of mesourban districts on the base case, which facilitates greater improvements on the obtained solutions. Then, when comparing the Robust and FLEET-EXC model, there is a slight advantage for the

**Figure 5.5:** Comparison of experiments coverage mean difference between models.

**(a)** For all districts.




**(b)** For urban districts.




**(c)** For mesourban districts.

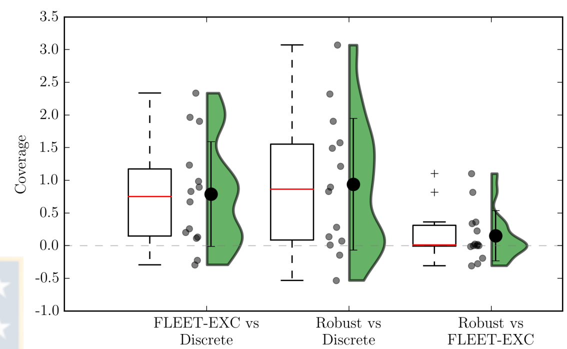



**Table 5.2:** Coverage results by type of emergency.

| Exp. | Discrete model | | | | FLEET-EXC model | | | | Robust model | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | Fire | Rescue | Hazmat | Forest fire | Fire | Rescue | Hazmat | Forest fire | Fire | Rescue | Hazmat | Forest fire |
| 1 | 30.72% | 52.38% | **30.47%** | **23.02%** | 30.72% | 52.42% | 30.38% | 23.01% | 30.72% | **52.43%** | **30.47%** | 23.01% |
| 2 | 32.46% | 51.86% | 29.59% | 23.62% | 32.09% | **52.55%** | **29.9%** | 23.53% | **33.03%** | 52.2% | 29.67% | **24.29%** |
| 3 | 45.08% | 56.52% | 30.1% | 23.27% | **45.34%** | **56.53%** | 30.1% | **23.28%** | **45.34%** | **56.53%** | 30.1% | **23.28%** |
| 4 | **53.1%** | 60.54% | **36.4%** | 26.19% | 52.95% | **62.23%** | 34.85% | **26.2%** | **53.1%** | **62.23%** | 34.26% | **26.2%** |
| 5 | 44.77% | 58.97% | 30.6% | **25.33%** | **48.55%** | **59.0%** | **30.79%** | 23.52% | **48.55%** | **59.0%** | **30.79%** | 23.52% |
| 6 | 53.91% | 60.74% | **37.66%** | **26.58%** | 54.5% | **63.35%** | 35.49% | 25.71% | **55.34%** | 62.3% | 34.3% | 25.88% |
| 7 | 47.78% | 56.6% | 30.25% | **28.97%** | **49.42%** | **58.95%** | **30.56%** | 27.93% | **49.42%** | **58.95%** | **30.56%** | 27.93% |
| 8 | 52.63% | 59.28% | 34.56% | **30.17%** | **54.68%** | 59.55% | 34.66% | 27.68% | 55.4% | **59.58%** | **34.81%** | 27.43% |
| 9 | 52.47% | 62.09% | **35.37%** | **29.5%** | 53.01% | 62.17% | 34.89% | 29.31% | **53.16%** | **62.40%** | 34.91% | 27.75% |
| 10 | 61.66% | 65.09% | 39.49% | 33.28% | 60.55% | 67.21% | 40.44% | **35.8%** | **61.95%** | **67.33%** | **40.63%** | 34.01% |
| 11 | 52.9% | 60.62% | **36.4%** | **30.32%** | 53.64% | 61.35% | 35.63% | 29.58% | **54.34%** | **61.56%** | 34.93% | 27.88% |
| 12 | 62.73% | 64.91% | 42.85% | **36.56%** | 62.13% | 65.01% | **43.06%** | 36.45% | **63.9%** | **67.4%** | 41.15% | 35.85% |
| 13 | 52.53% | 62.56% | **35.52%** | 29.3% | **56.74%** | **62.84%** | 34.18% | 29.43% | 56.63% | 62.83% | 34.79% | **29.53%** |
| 14 | 59.62% | 63.35% | **45.49%** | 34.01% | 62.87% | 66.41% | 39.09% | **38.67%** | **64.22%** | **68.52%** | 44.4% | 34.4% |

first one due to a positive average mean difference, although the median is 0%.

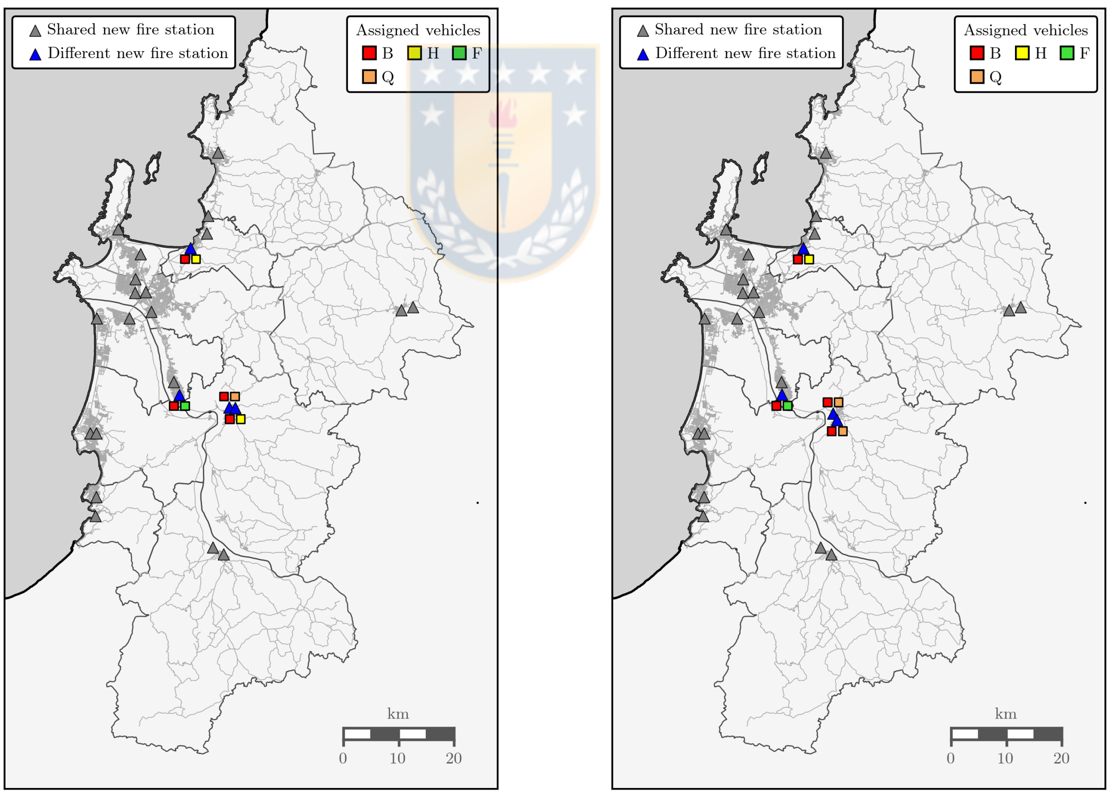
Furthermore, the coverage of emergencies per type is presented in Table 5.2 for all experiments, where the highest coverages for the three models are highlighted. As can be seen, the minor overall coverage of the Discrete model is explained by the higher priority

**Table 5.3:** New vehicle assignments.

| Exp. | Discrete model | | | | | FLEET-EXC model | | | | | Robust model | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| # | B | Q | R | H | F | B | Q | R | H | F | B | Q | R | H | F |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 12 | 7 | 5 | 0 | 0 | 12 | 7 | 5 | 0 | 0 | 12 | 7 | 5 | 0 | 0 |
| 4 | 24 | 11 | 7 | 3 | 3 | 24 | 11 | 8 | 2 | 3 | 24 | 12 | 8 | 1 | 3 |
| 5 | 12 | 7 | 4 | 0 | 1 | 12 | 8 | 4 | 0 | 0 | 12 | 8 | 4 | 0 | 0 |
| 6 | 24 | 10 | 7 | 4 | 3 | 24 | 12 | 8 | 2 | 2 | 24 | 11 | 9 | 2 | 2 |
| 7 | 12 | 7 | 3 | 0 | 2 | 12 | 7 | 4 | 0 | 1 | 12 | 7 | 4 | 0 | 1 |
| 8 | 24 | 11 | 6 | 3 | 4 | 24 | 12 | 6 | 3 | 3 | 24 | 12 | 6 | 3 | 3 |
| 9 | 24 | 10 | 8 | 2 | 4 | 24 | 11 | 8 | 1 | 4 | 24 | 12 | 8 | 1 | 3 |
| 10 | 48 | 21 | 12 | 6 | 9 | 48 | 20 | 12 | 5 | 11 | 48 | 20 | 13 | 6 | 9 |
| 11 | 24 | 10 | 7 | 3 | 4 | 24 | 11 | 7 | 2 | 4 | 24 | 12 | 8 | 1 | 3 |
| 12 | 48 | 20 | 10 | 7 | 11 | 48 | 19 | 11 | 8 | 10 | 48 | 20 | 11 | 7 | 10 |
| 13 | 24 | 10 | 8 | 3 | 3 | 24 | 12 | 8 | 1 | 3 | 24 | 11 | 8 | 2 | 3 |
| 14 | 48 | 20 | 10 | 9 | 9 | 48 | 20 | 12 | 5 | 11 | 48 | 20 | 11 | 7 | 10 |

**Figure 5.6:** Fire station and vehicle assignments decisions comparisons between the Discrete and FLEET-EXC models for experiment 9.

**(a)** Discrete model. **(b)** FLEET-EXC model.




on covering hazmat and forest fire emergencies, which are less frequent than urban fires and rescue. As a consequence, more $H$ and $F$ vehicles are assigned to the newer fire stations in detriment of $Q$ and $R$ vehicles as shown in Table 5.3. Additionally, because fire and rescue emergencies are more frequent, the FLEET-EXC and Robust models assign these types of specialty vehicles to diminish their average utilization and maximize the overall coverage. As a result, $Q$ vehicles are the most assigned specialty vehicles on all experiments, followed by $R$ ones.

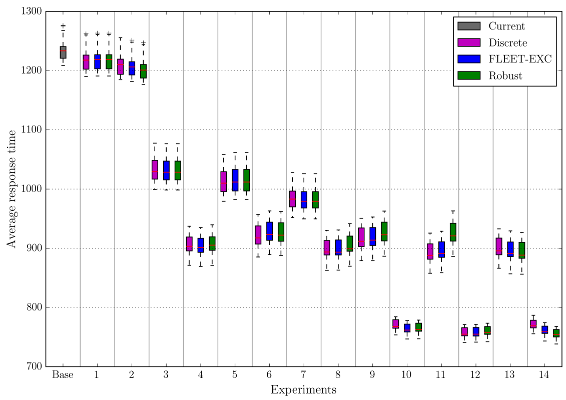
The aforementioned difference in coverage between models is a consequence of different decisions on fire station locations and vehicle assignments. Figure 5.6 shows the optimal solutions of both Discrete and FLEET-EXC models for experiment 9, where the second one has a higher overall coverage of emergencies. The main differences consist of the location of new facilities with $H$ and $F$ specialty vehicles on districts 6 and 8 respectively, and the assignment of a $Q$ vehicle on the FLEET-EXC instead of the $H$ vehicles assigned on the Discrete model. The assignment of vehicles per district is shown in appendix A.

## 5.4 Response time results

Another performance parameter to consider in the analysis is the response time of the entire fleet of dispatched vehicles. Figure 5.7 shows a comparison of the average response time between models for all experiments. First, Figure 5.7(a) presents the average response time considering all emergencies, where the base case average response time is above 1200 seconds. The results divided for each emergency type is presented in appendix D. As expected, lower average response time is achieved when we add more resources to the system, reaching values below 800 seconds on experiments 10, 12 and 14. On the other hand, contrary to the results obtained for the coverage of emergencies, there is no trend towards a higher improvement of the FLEET-EXC and Robust models compared to the Discrete model across experiments. Moreover, Figure 5.7(b) presents the same comparison but only considering the emergencies attended below the 60th percentile of response times. We considered this range of values to analyze the performance of the model with ´near-to-be-covered' emergencies. The figure shows that the Robust model has lower average response time, which is expected since this model optimizes the worst case of emergency coverage and, therefore, can handle better the stochasticity of simulated replicas. Furthermore, Figure 5.8(a) shows the mean difference of average response time

**Figure 5.7:** Average response time comparison between experiments.

(a) All emergencies.




(b) Emergencies with response time below 60th percentile.

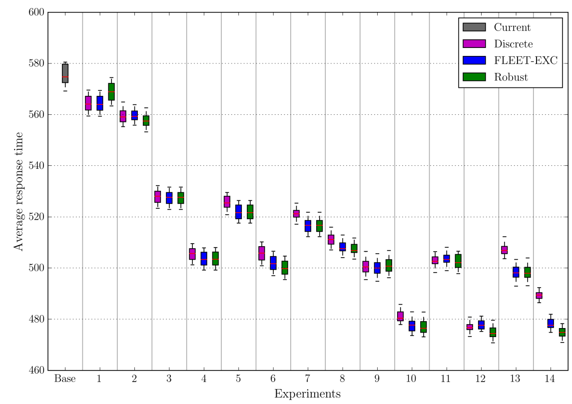
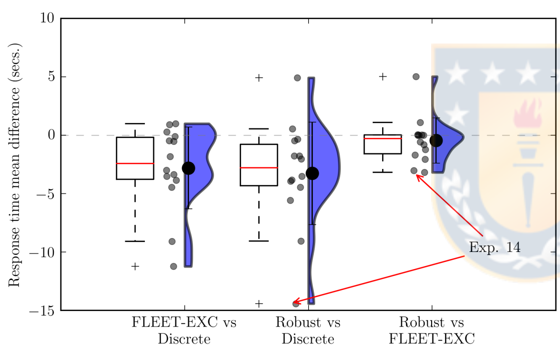
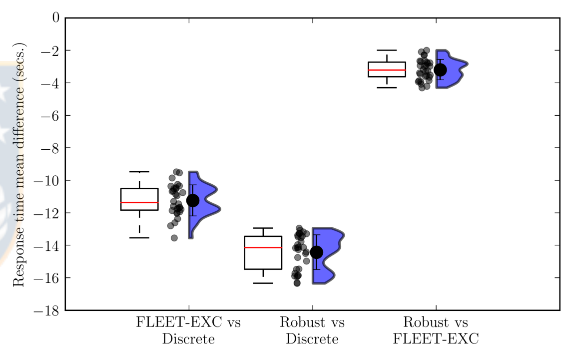



**Figure 5.8:** Comparison of mean difference on response time below 60th percentile.

(a) Between models considering all experiments.




(b) Between replicas in experiment 14.




between models across all experiments, where the Robust model performs better than the FLEET-EXC and Discrete models. Additionally, this difference is broader when more changes are allowed in the system. For example, Figure 5.8(b) shows the robust model has a statistical difference in the response time with the other models when comparing replicas for experiment 14.

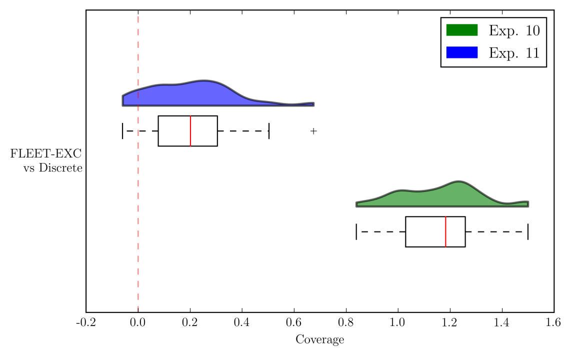
This is explained because the proposed models maximize the coverage of emergencies instead of minimizing their response time. Although there exists a relationship between the response time and the coverage of emergencies, lower response time is not necessarily translated into a higher coverage, and vice-versa. To explain this relationship, we compared the coverage and response time results for experiments 10 and 11. Both experiments resulted in higher coverage for the FLEET-EXC model in comparison with the discrete one, as shown in Figure 5.9(a). On the other hand, Figure 5.9(b) presents the mean difference in response time between both models. Even though on experiment 10 the FLEET-EXC has better coverage, the resulting solution has a worst average response time when compared to the solution from the Discrete model. On the contrary, the FLEET-EXC has a better response time performance for experiment 11. This is mainly explained by the distribution of the response times, as shown in Figure 5.10, where the difference of the density function of both models is presented for urban and mesourban emergencies. The difference is represented as the subtraction between the distribution of the response time results from the FLEET-EXC and the Discrete model; therefore, if the density difference is positive over a time interval, the FLEET-EXC results have a greater number of emergencies attended on that range of response times. For example, if the cumulative density difference is positive for $x = t$, the FLEET-EXC results have a greater number of emergencies served under or equal to $t$ than the Discrete model results. Thus, this metric illustrates the coverage difference between models when $t$ equals the response time target $t_{max}$. Figure 5.10(a) and Figure 5.10(b) show that both urban and mesourban emergencies have higher coverage for the FLEET-EXC results. For the urban emergencies, although the Discrete model achieves more events served under 350 seconds, the FLEET-EXC results have a greater amount of emergencies served under the response time target of 600 seconds. Furthermore, the FLEET-EXC results have more events attended over 7000 seconds for mesourban districts. As a consequence of these results, the response time average of the FLEET-EXC results is higher than the Discrete ones, while maintaining a higher coverage of emergencies. Finally, we can conclude that an increase in coverage does not necessarily imply a lower average response time, and the proposed model can produce an increment of undesired long times in the response of emergencies.

## 5.5 Base Case v/s Experiments

In the following subsection, two experiments were selected to perform a more thorough analysis of the solutions obtained with the FLEET-EXC model compared with the current layout of fire departments. The FLEET-EXC was chosen over the others because it computes the ´average' solution, while the robust and discrete models calculate the worst and

**Figure 5.9:** Comparison of mean differences for coverage and response time between experiments 8, 10 and 11.

**(a)** Coverage.




**(b)** Response time.

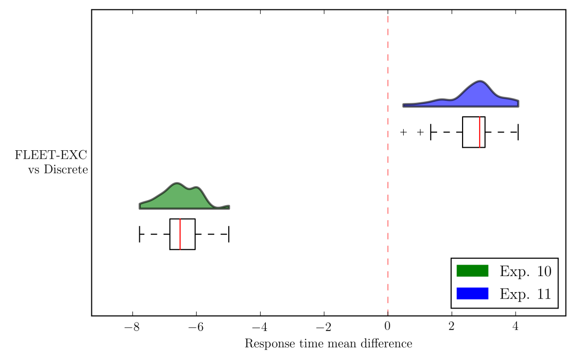
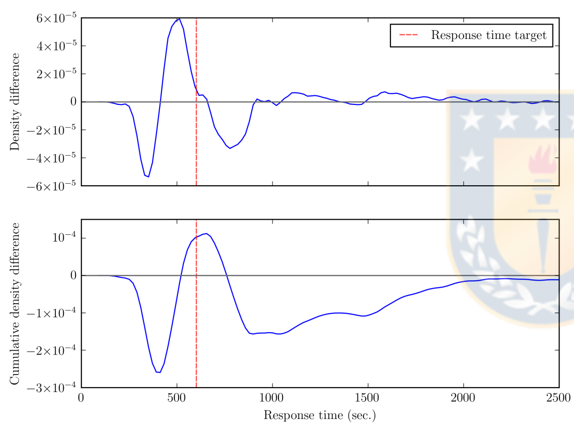
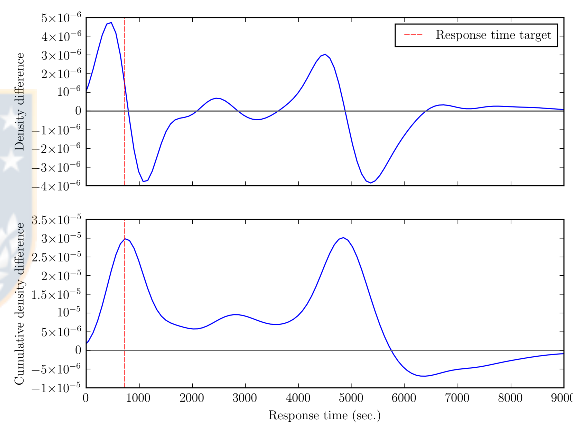



**Figure 5.10:** Response time density distribution difference between FLEET-EXC and Discrete models for experiment 11.

**(a)** Urban districts emergencies.



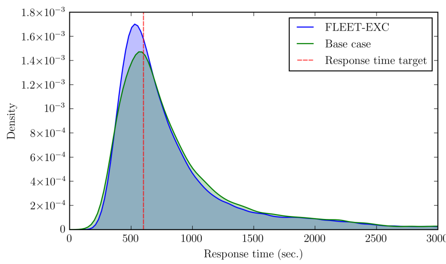

**(b)** Mesourban districts emergencies.




best case scenarios respectively. Experiments 1 and 3 were selected because they are the most likely improvements that each fire department can make. Additionally, a new experiment was developed with no constraints on either where to locate fire stations or which vehicle to assign. The only consideration was that the vehicles of a fire department cannot be assigned to another. The goal was to quantify the impact of optimization models for resource allocation on urban planning. First, Figure 5.11 shows the response time density improvement of relocating a single fire station on each district, where higher coverage is achieved for both urban and mesourban areas. This increment in coverage is primarily due to the relocation of fire stations with $B$ and $Q$ vehicles to better serve fire and rescue emergencies. Figure 5.12 shows the variation in the coverage of fire emergencies and the relocated facilities with their respective vehicles. It also illustrates that the new locations of fire stations tend to be close to inter-district roads, facilitating the cooperation between fire departments.

**Figure 5.11:** Response time density comparison between FLEET-EXC model results for experiment 1 and the base case.

**(a)** Urban districts emergencies.




**(b)** Mesourban districts emergencies.

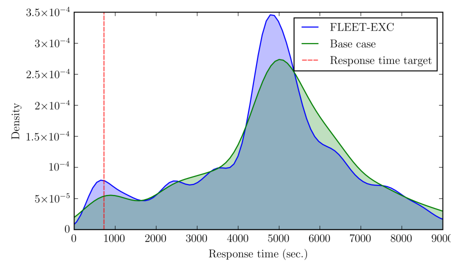
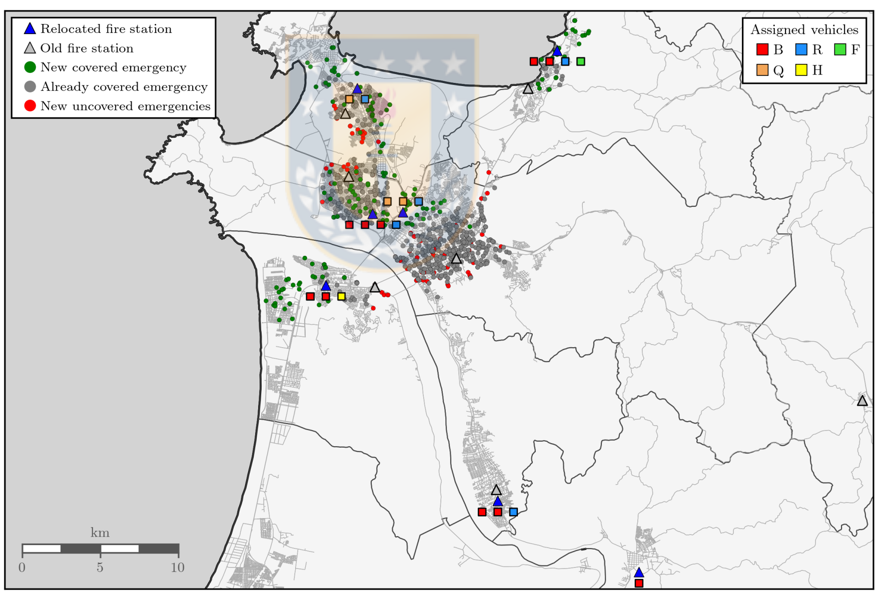


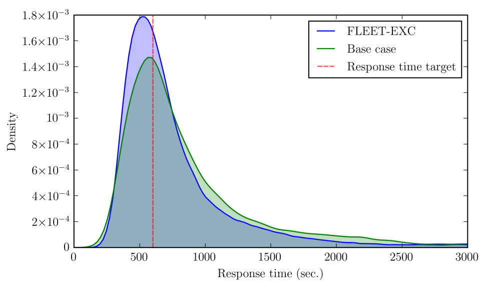

**Figure 5.12:** Fire emergencies coverage improvement of the FLEET-EXC model optimal solution on experiment 1.




When comparing the response time results from experiment 3 (one new facility and one extra basic and specialty vehicles) with the base case on Figure 5.13, there is a much higher improvement on coverage. More specifically, Figure 5.13(b) shows that the density distribution of response times on mesourban districts change substantially. This increase is explained by the fact that currently there is a deficit of specialty vehicles on these fire department, therefore, by assigning the most required vehicle a greater marginal improvement on coverage is made. On the other hand, the coverage of urban districts emergencies has a lower improvement, affecting mostly the coverage of fire and rescue emergencies. Figure 5.14 shows the improvement of fire emergencies and the location of the new fire stations.

**Figure 5.13:** Response time density comparison between FLEET-EXC model results for experiment 3 and the base case.

**(a)** Urban districts emergencies.




**(b)** Mesourban districts emergencies.

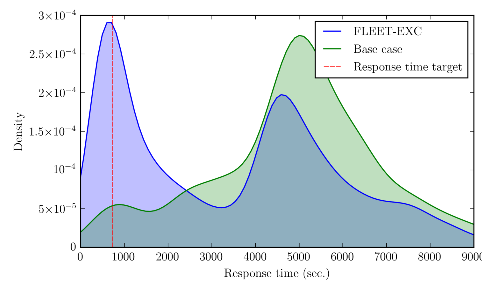
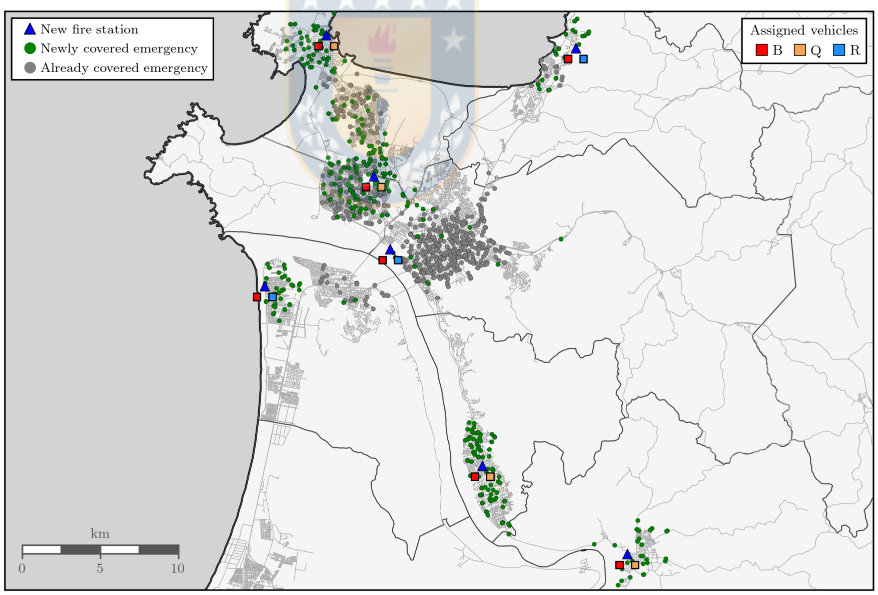


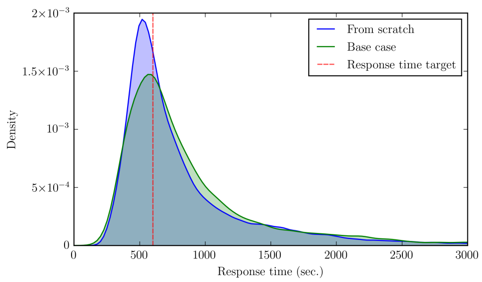

**Figure 5.14:** Fire emergencies coverage improvement of the FLEET-EXC model optimal solution on experiment 3.




Evaluating the planning of fire departments' layout from scratch gives a glance at how the emergency system would perform when available resources are used optimally. Figure 5.15 shows that the major improvement is made on the coverage of emergencies of urban districts. Furthermore, this is clearly depicted on Figure 5.16, where the coverage of fire emergencies is shown for both the base case and the planning-from-scratch scenario. Figure 5.16(b) depicts a more homogeneous distribution of fire stations with type $Q$ vehicles on the fire department of district 1, which increases the fire emergency coverage in 6.77%. Moreover, an overall gain of 4.79% on emergency coverage is achieved using the same resources currently available.

**Figure 5.15:** Response time density comparison between planning-from-scratch scenario and the base case.

**(a)** Urban districts emergencies.




**(b)** Mesourban districts emergencies.

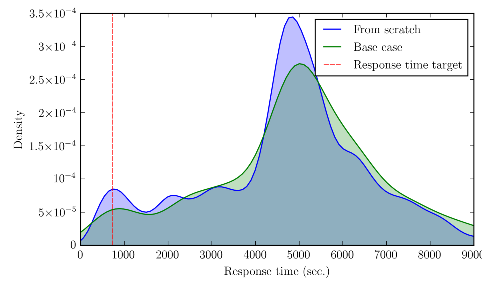
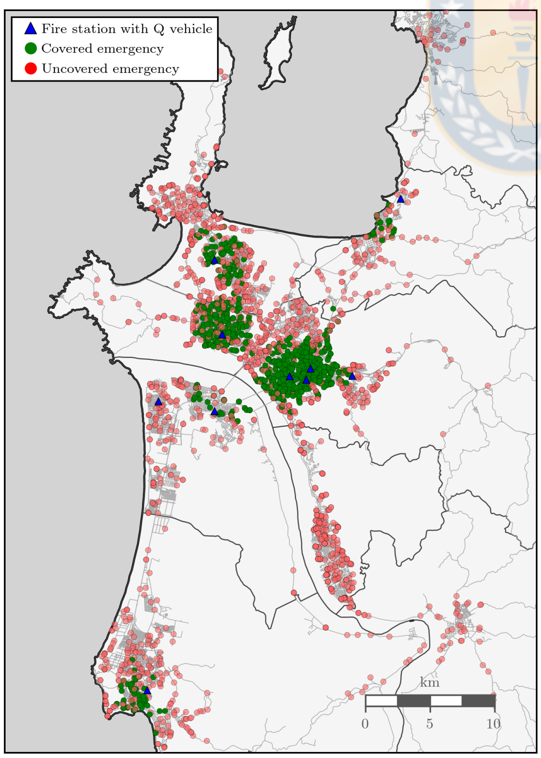
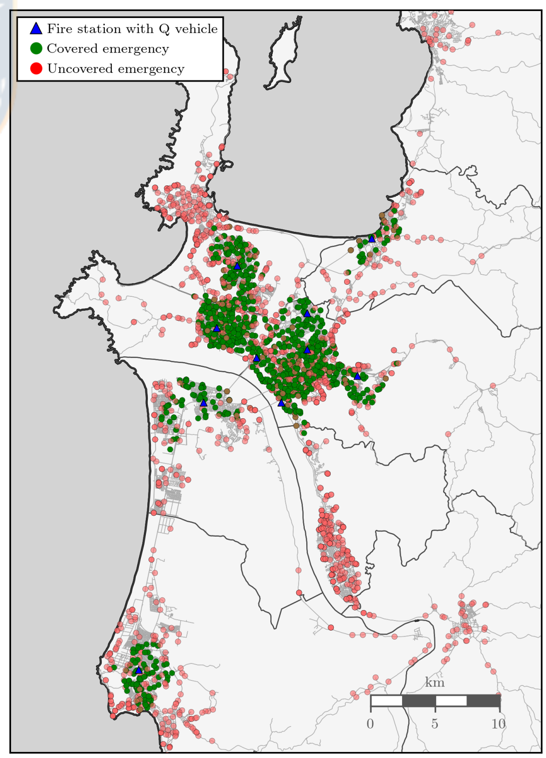



**Figure 5.16:** Fire emergencies coverage comparison between the base case and planning-from-scratch scenario.

**(a)** Base case.




**(b)** From scratch.



## 5.6 Insights

To summarize, the main insights from the obtained results are:

- The proposed spatio-temporal sampling method outperforms the ones commonly used in literature when replicating the behavior of emergencies.
- When comparing experiments with the same number of resources allocated, e.g. experiments 4 and 8 with four new vehicles per fire department (two specialty and two basic) or experiments 6 and 11 with the same number of vehicle assignments but with a single relocation, the overall coverage of emergencies is similar. Based on this, planners could be prone to assign more vehicles instead of locating new facilities, due to the fact that opening a fire station is more difficult than buying a new vehicle.
- The discrete FLEET model prioritizes low-frequency emergencies (forest fires and hazmat types) due to the fact that it does not account for the higher utilization of $Q$ and $R$ vehicles to attend fire and rescue emergencies. On the other hand, the robust model by optimizing the worst-case scenario of coverage increases the amount of $Q$ and $R$ vehicles to assign on new fire stations.
- The Robust model performed better when more changes are allowed in the system, although currently, these scenarios are unrealistic.
- Considering the above, the FLEET-EXC model should be preferred by current fire departments' planning teams to avoid the overpopulation of a certain type of vehicle and balance the purchase of new vehicles.
- The location of new fire stations and the relocation of current ones tends to be near inter-districts roads to facilitate the cooperation between fire departments by complementing the scarcity of specialty vehicles.
- The Discrete model seems to be more robust than expected, achieving a similar performance with the other models in some experiments. Although, this may be produced due to the small number of candidates nodes on some districts, leading to

similar location solutions between models, and the limited number of changes that can be done in the system due to policy issues.

- An increase in coverage does not necessarily imply a lower average response time, which might be undesirable by planning teams and must be considered when making location decisions.

# Chapter 6

# Conclusions and extensions

In this work, the facility location and vehicle assignment problem was revisited for firefighters operations, where multiple emergency types must be attended using a district-dependent dispatch policy. To solve this strategic decision-making problem, an iterative simulation-optimization approach was proposed, that at each iteration updates the optimal location of vehicles and fire stations based on the utilization parameters computed in the simulation model. Moreover, to increase the realism of the realization of emergencies, a two-stage spatio-temporal distribution was proposed, where the emergency arrival process is modeled by a KDE for the spatial component and a Markov-MECO for the temporal one. For the optimization part of the procedure, three models were developed: a Discrete FLEET model, the FLEET-EXC model, and a Robust FLEET-EXC model.

The main results show that spatio-temporal sampling method that uses an NSNR arrival process and a KDE has a better representation of the emergencies arrival process than the ones generally used in literature, mainly because: i) it takes into account correlation between interarrival times increasing the utilization of vehicles, ii) it assumes a non-renewal distribution which is more realistic than Poisson distribution, and iii) it smoothly adjusts a spatial distribution that diminishes the error associated with aggregating events into quadrants. Moreover, the simulation-optimization procedure that uses the FLEET-EXC model and its Robust counterpart have higher emergency coverage than the Discrete FLEET model on most of the proposed scenarios. This is because the Discrete FLEET assigns a higher priority to cover lower frequency events than the other models. Moreover, because the discrete FLEET model optimizes the best-case scenario of coverage (with full availability of vehicles) and the Robust model the worst case, the results of the FLEET-EXC model should be considered by planners as a more balanced vehicles assignment solution. Additionally, relocation of fire stations near inter-districts roads may help to support the coverage of uncovered emergencies due to the lack of specialty vehicles. The proposed DES model accurately represents the arrival and attendance of emergencies, which can be replicated to perform online optimization on similar ESS related problems such as ambulances redeployment. Nevertheless, improvements can be made by using a multidimensional bandwidth for the kernel density estimator in order to better fit the spatial distribution. Finally, as a future study we want to include cost estimates in the objective function by using a multi-objective approach.

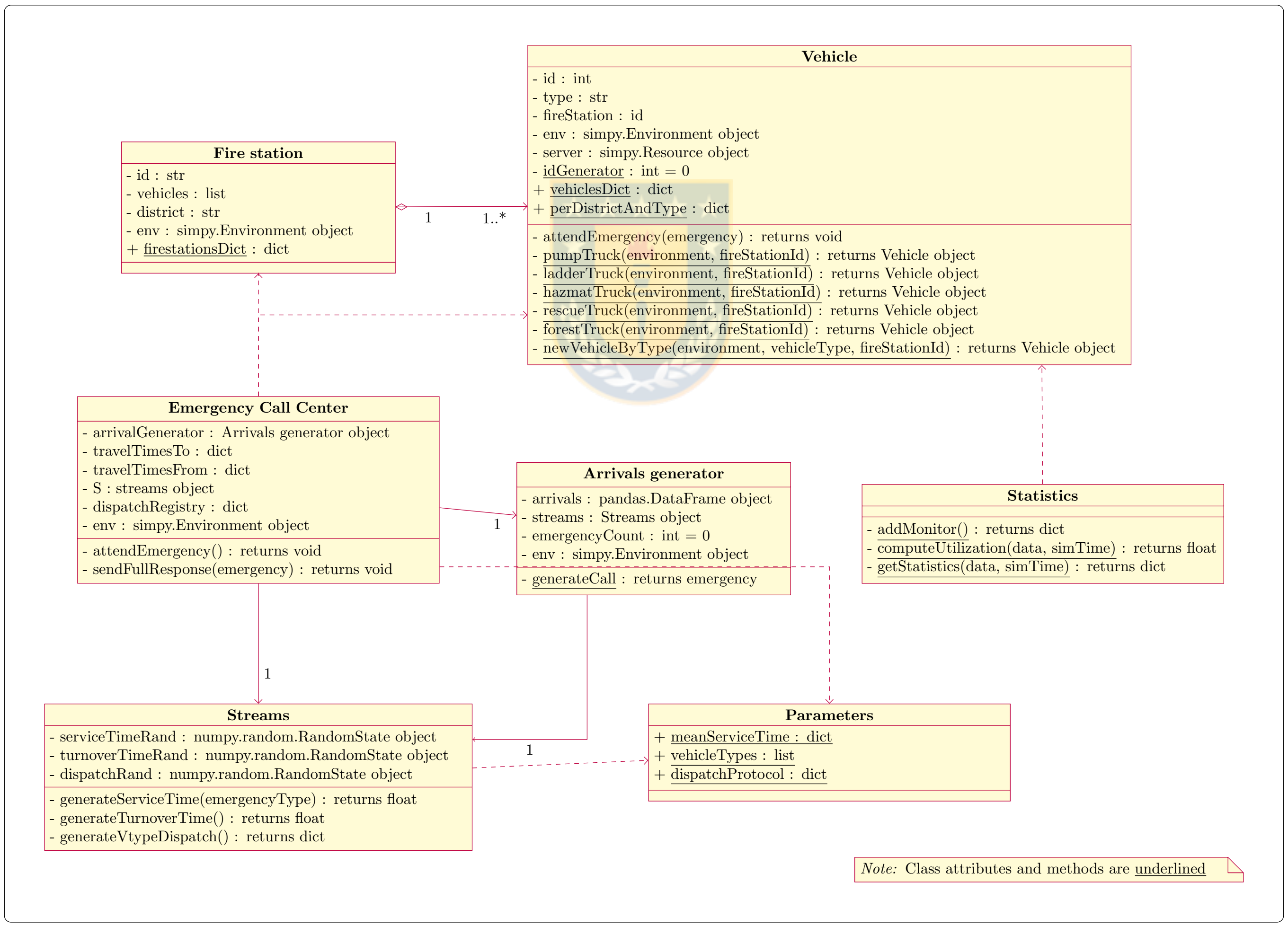
# Appendix

## A Vehicle assignment per district.

| | | Vehicle assignment per district | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | | | | 2 | | | | | 3 | | | | | 4 | | | | | 5 | | | | | 6 | | | | | 7 | | | | | 8 | | | | | 9 | | | | | 10 | | | | | 11 | | | | | 12 | | | | |
| Experiment | Model | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f | b | q | r | h | f |
| | discrete | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | fleet | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | robust | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | discrete | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | fleet | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | robust | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | discrete | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 3 | fleet | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| | robust | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| | discrete | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| 4 | fleet | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | robust | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 |
| | discrete | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 5 | fleet | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| | robust | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| | discrete | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 0 | 0 | 1 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| 6 | fleet | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | robust | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | discrete | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 7 | fleet | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| | robust | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| | discrete | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| 8 | fleet | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | robust | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | discrete | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 0 | 1 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| 9 | fleet | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 0 | 1 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 |
| | robust | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 |
| | discrete | 4 | 1 | 2 | 0 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 0 | 2 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 2 | 0 | 4 | 1 | 1 | 1 | 1 |
| 10 | fleet | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 0 | 2 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 2 | 0 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 1 | 1 | 1 |
| | robust | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 1 | 1 | 1 |
| | discrete | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| 11 | fleet | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | robust | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 |
| | discrete | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 0 | 2 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 1 | 0 | 2 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 1 | 1 | 1 |
| 12 | fleet | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 1 | 0 | 2 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 1 | 1 | 1 |
| | robust | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 0 | 2 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 1 | 1 | 1 |
| | discrete | 2 | 0 | 1 | 0 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| 13 | fleet | 2 | 0 | 2 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 0 | 0 |
| | robust | 2 | 0 | 2 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 1 | 0 | 0 |
| | discrete | 4 | 1 | 0 | 2 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 0 | 2 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 2 | 0 | 4 | 2 | 0 | 2 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 1 | 1 | 1 | 1 |
| 14 | fleet | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 0 | 0 | 2 | 4 | 2 | 0 | 0 | 2 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 1 | 0 | 2 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 2 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 1 | 1 | 0 | 4 | 1 | 2 | 0 | 1 |
| | robust | 4 | 1 | 1 | 1 | 1 | 4 | 3 | 1 | 0 | 0 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 1 | 0 | 2 | 4 | 2 | 1 | 0 | 1 | 4 | 1 | 2 | 1 | 0 | 4 | 2 | 0 | 2 | 0 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 1 | 1 | 1 | 4 | 2 | 1 | 0 | 1 | 4 | 2 | 0 | 1 | 1 | 4 | 1 | 2 | 0 | 1 |

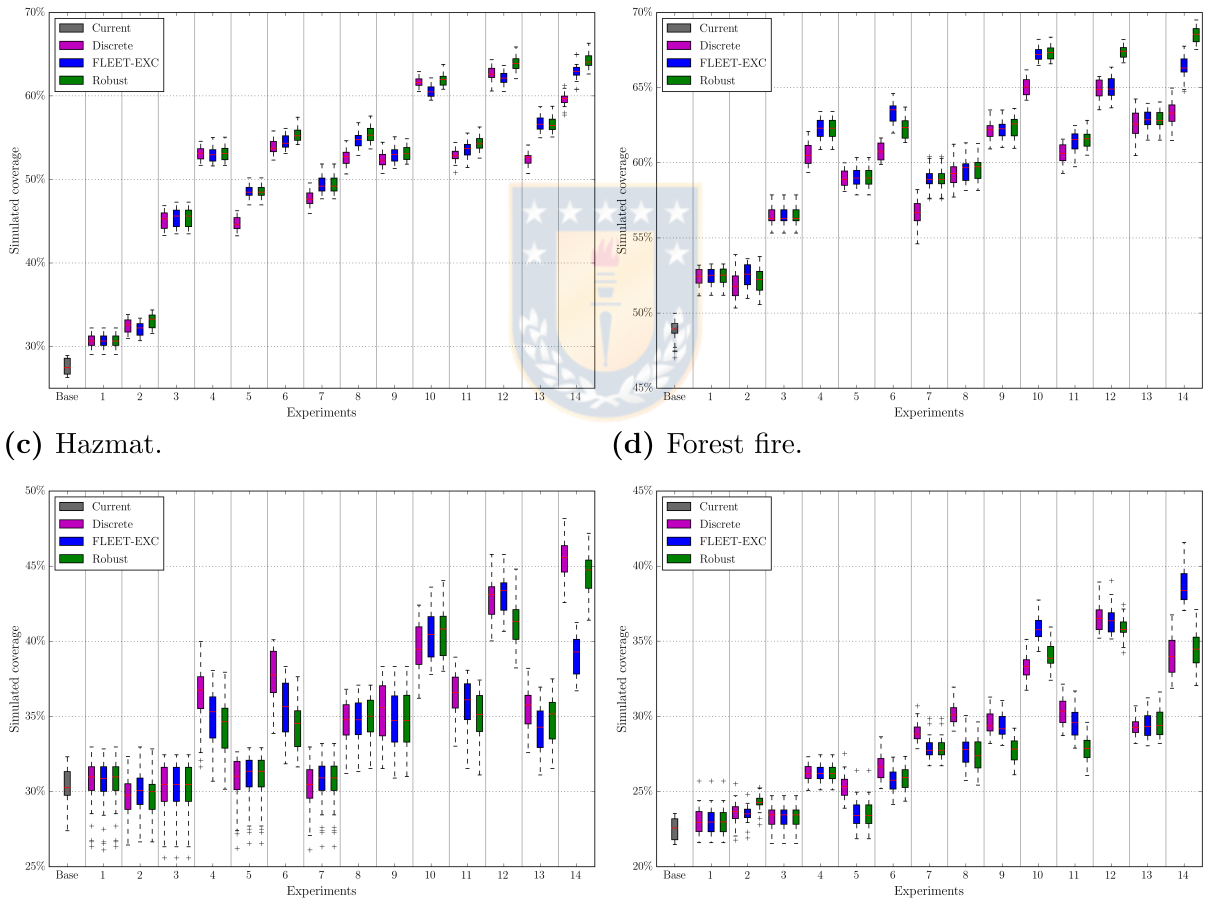
# B UML diagram of the simulation model.

**Figure 6.1:** UML diagram.



# C Coverage results per experiment.

**Figure 6.2:** Coverage comparison between experiments per emergency type.




(a) Fire.

(b) Rescue.

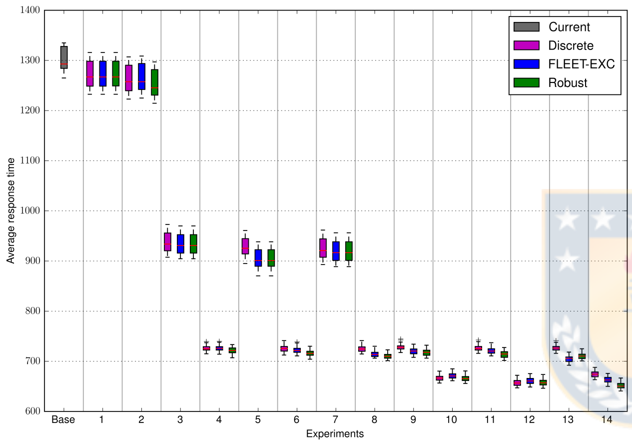
(c) Hazmat.

(d) Forest fire.

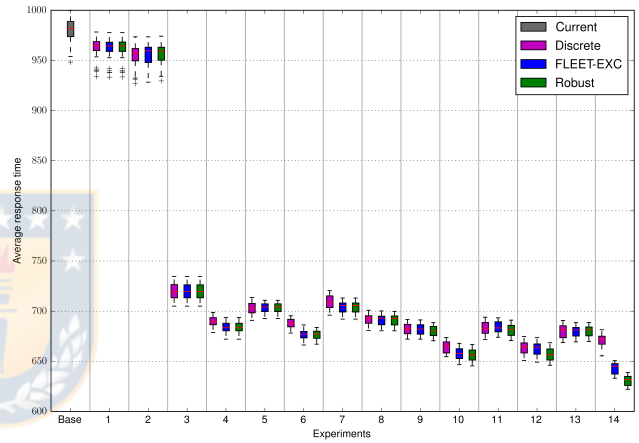
# D Response time results per experiment.

**Figure 6.3:** Response comparison between experiments per emergency type.

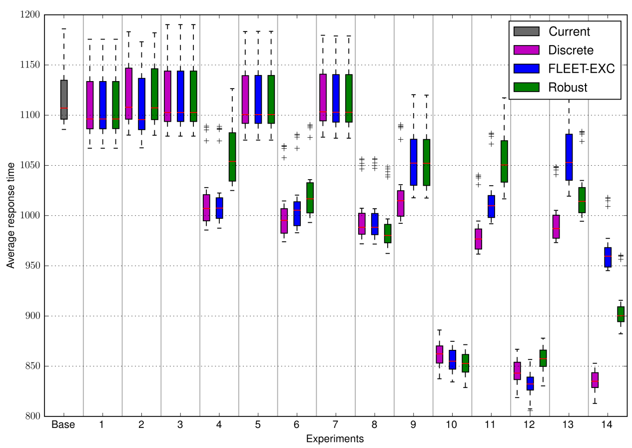
**(a)** Fire.



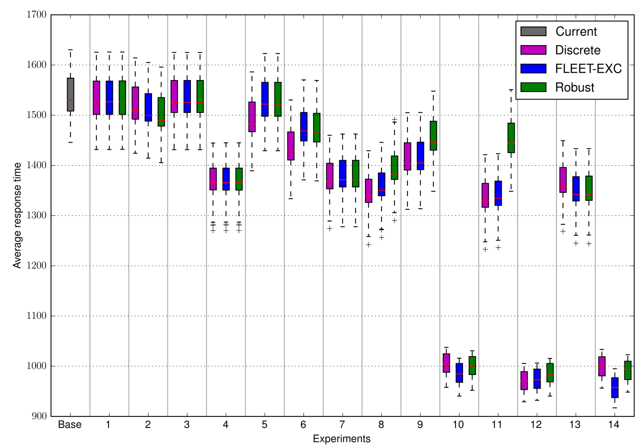

**(b)** Rescue.




**(c)** Hazmat.




**(d)** Forest fire.



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